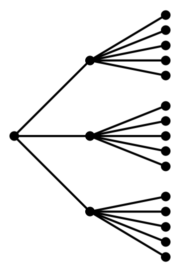
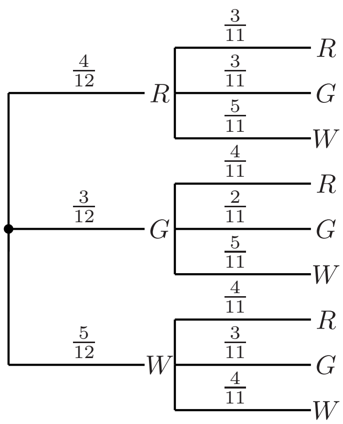
Review for Exam 3
Math 102
Answers

Complete the following exercises for a review. Questions on the exam will be similar to questions on this review, questions from the homework assignments and the suggested exercises.

1. There are 15 branches in this tree.



2. There are $4 \times 4 \times 2 = 32$ branches.
3. There are $8 \times 8 \times 8 \times 5 \times 5 \times 5 \times 5 = 40,000$ ways.
4. There are $C(12,3) = 220$ ways.
5. There are $15 \times 14 = 210$ ways.
6. There are $C(15,3) = 455$ ways.
7. There are $15 \times 14 \times C(13,2) = 16380$ ways.
8. Suppose you have an urn with 4 red balls, 3 green balls and 5 white balls.
   (a) $C(12,4) = 495$ ways.
   (b) $4 \times C(5,3) = 40$ ways.
   (c) $C(3,2) \times C(5,2) = 30$ ways.
   (d) $C(5,4) = 5$ ways.
9. Suppose you are in a theatre and there are 6 seats in a row.
   (a) $P(6,6) = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways.
   (b) $P(15,6) = 3,603,600$ ways.
   (c) Together: $5 \times 2 \times P(4,4) = 240$; Separate: $720 - 240 = 480$.
10. $4 \times C(13,3) \times 3 \times C(13,2) = 267696$n ways.
11. $13 \times C(4,4) \times 48 = 624$
12. $\frac{13 \times C(4,3) \times 12 \times C(4,3)}{2} = 1248$ ways.
13. $12 \times 11 \times 10 \times 9 = 11880$; $C(12,4) = 495$; $C(12,2) \times 4! = 1584$
14. (b): $\frac{40}{495} = \frac{8}{99} \approx 0.0808$
    (c): $\frac{30}{495} = \frac{2}{33} \approx 0.0606$
    (d): $\frac{5}{495} = \frac{1}{99} \approx 0.0101$



15.

(a) $P(R_1|R_2) = \frac{P(R_1 \cap R_2)}{P(R_1)} = \frac{1/11}{1/3} = \frac{3}{11} \approx 0.2727$

(b) $P(R_1 \cap G_2) + P(G_1 \cap G_2) + P(W_1 \cap G_2) = \frac{1}{4}$

(c) $P(R_1|G_2) = \frac{P(R_1 \cap G_2)}{P(G_2)} = \frac{1/11}{1/4} = \frac{4}{11}$

16. $\frac{2}{36} = \frac{1}{18}$

17. $\frac{17}{18}$

18. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{22}{52} \approx 0.423$

19. The following table classifies 1456 people by their sex and whether or not they favor a gun law. Use it to answer the questions below.

	Male (S_1)	Female (S_2)	Totals
Favor (A_1)	392	649	1041
Oppose (A_2)	241	174	415
Totals	633	823	1456

(a) $P(A_2|S_1) = \frac{P(A_2 \cap S_1)}{P(S_1)} = \frac{241}{633} \approx 0.38$

(b) $P(S_1|A_1) = \frac{P(S_1 \cap A_1)}{P(A_1)} = \frac{392}{1041} \approx 0.38$

(c) $P(S_2|A_1) = \frac{P(S_2 \cap A_1)}{P(A_1)} = \frac{649}{1041} \approx 0.62$

(d) $P(A_1|S_2) = \frac{P(A_1 \cap S_2)}{P(S_2)} = \frac{649}{823} \approx 0.79$

20. E: less than 5. F: odd. $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = 1/9$. $P(E) = 1/6$, so $P(E) \neq P(E|F)$, so the events are dependent.

21. Without Replacement: $P(K_2|J_1) = \frac{P(K_2 \cap J_1)}{P(J_1)}$. We need to find each of the probabilities in that expression. $P(K_2 \cap J_1) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{633}$, and $P(J_1) = \frac{4}{52} = \frac{1}{13}$. Thus, $P(K_2|J_1) = \frac{4/633}{4/52} = \frac{52}{633} \approx 0.0821$.

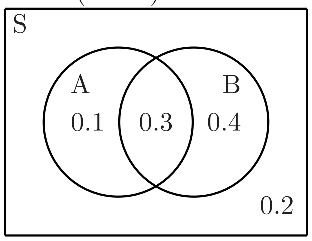
With Replacement: $P(K_2|J_1) = \frac{P(K_2 \cap J_1)}{P(J_1)}$. This time, $P(K_2 \cap J_1) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$. So $P(K_2|J_1) = \frac{1/169}{4/52} = \frac{1}{13}$.

We want to know if the events drawing a Jack on the first draw and drawing a King on the second draw are independent. We'll determine the probability that a King is drawn on the second draw. First we do this without replacement. We could have one of two things happen on the first draw. We could either draw a king or we could not draw a king. Let K_1 be the event that we draw a king and

K_1' be the event that we do not draw a king. Then $P(K_2) = P(K_2|K_1)P(K_1) + P(K_2|K_1')P(K_1') = \frac{48}{52} \cdot \frac{4}{51} + \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13}$. Since, $P(K_2) \neq P(K_2|J_1)$ these events are dependent.

Now, we compute $P(K_2)$ with replacement. $P(K_2) = \frac{4}{52} = \frac{1}{13}$. This is the same as $P(K_2|J_1)$, so these events are independent.

22. Suppose that you have sample space S and events A and B. Suppose that $P(A) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$. Draw a probability Venn diagram and use it to find the following probabilities.



(a) 0.8	(d) 0.3	(g) 0.7
(b) 1	(e) 0.2	(h) 0.4
(c) 0.6	(f) 0.1	(i) 0.4

23. Either you win or you lose. If you win you get \$499. If you lose, you lose \$1. The expected value is $\frac{1}{1000} \cdot 499 + \frac{999}{1000} \cdot (-1) = \frac{499-999}{1000} = -0.5$. You can expect to lose 50 cents.

24. The expected value of this game is $\frac{3}{6}(-2) + \frac{1}{6}(10) + \frac{1}{6}(-1) + \frac{1}{6}(2) = \frac{-6+10-1+2}{=}\frac{5}{6} \approx 0.833$. You can expect to gain 83 cents. The game is not fair. You have an advantage.

25. In this case a success is a head and a failure is a tail. The probability of a success is $\frac{1}{2}$ and the probability of a failure is $\frac{1}{2}$. We are performing the experiment 12 times. We want to have 6 successes. Using the binomial theorem we get that the probability is $C(12,6)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^6 \approx 0.2256$.

    In the next experiment a success is a tail and a failure is a head. The probability of a success is $\frac{1}{2}$ and the probability of a failure is $\frac{1}{2}$. We are performing the experiment 12 times. We want to have 2 successes. Using the binomial theorem we get that the probability is $C(12,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 0 \approx 0.0161$.

26. In the first case a success is a 5 or 6 and a failure is anything else. The probability of a success is $\frac{1}{3}$ and the probability of a failure is $\frac{2}{3}$. We are performing the experiment 8 times and want 3 successes. The probability of this is $C(8,3)\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5 \approx 0.2731$.

    In the next case a success is an even number and a failure is an odd number. The probability of a success is $\frac{1}{2}$ and the probability of a failure is $\frac{1}{2}$. We are performing the experiment 8 times. We want to have 5 successes. Using the binomial theorem we get that the probability is $C(8,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 \approx 0.2188$.