MATH 127 Review Sheet for Final Exam

COMMENT: Do **NOT** try to do every problem. It would not be a good studying strategy. Try at least 1 from each section. Do more if you are struggling. When you feel you have command over a certain set of problems, feel free to move on. This review sheet is a good representative of what problems will be like on the final. This does not mean that the problems on the final will be exactly like these, but the concepts will be the same.

1. Simplify the following expressions.

   a. $\left(\frac{1}{2}x^3\right)(8x^2)$ b. $\left(\frac{x^2y^{-4}}{xy^3}\right)$ c. $(3y^3z^2)(2y^{-5}zw^0)$

   d. $(2y)^3(5y)^2$ e. $\left(\frac{3x^3y^{-2}z}{15x^{-5}y^4z^3}\right)^2$ f. $\sqrt{2x^3y^4}$

   g. $\sqrt[3]{24x^4y^6z^{11}}$ h. $\frac{\sqrt{50x^7y^{11}}}{\sqrt{2x^3y^5}}$ i. $(3\sqrt[4]{4xyz})(2\sqrt[4]{4x^3y^4z^0})$

2. Write the expression using rational exponents.

   a. $\sqrt[4]{x^3}$ b. $\sqrt[5]{y^2}$ c. $\sqrt{(a+b)^5}$

3. Write the following using radicals.

   a. $x^{3/2}$ b. $y^{5/3}$ c. $(4+x)^{7/3}$

4. Perform the following Operation.

   (a) $(2x^2-3x+4)+(x^3+4x^2-1)$

   (b) $(3x^5+x^4-2x^2+x)+(8x^3-12x^2+4x-7)$

   (c) $(2x^4-x^2+44)-(6x^2-3x+2)$

   (d) $(23x^7+9x^4-x^3+22x)-(6x^8-x^7+2x^5+13x^4-x^2+x-9)$

   (e) $(x-3)(x+4)$

   (f) $(2x+1)(x-7)$

   (g) $(x+4)(2x^2+3x-1)$

   (h) $(x^2-1)(x^2+1)$

   (i) $(2x^4-4x^3+8x^2+14x)\div(2x)$

   (j) $(25x^4-15x^3+100x^2)\div(5x^2)$

5. Factor the following polynomials Completely.

   a. $x^2-6x-27$ b. $x^2+9x-22$ c. $3x^4-30x^3+63x^2$

   d. $2x^2+15x+7$ e. $9x^2+9x+2$ f. $2x^2+3x-9$

   g. $12x^2+14x-10$ h. $5x^3-6x^2-8$ i. x^3-8

   j. $9x^2-4$ k. x^2-16 l. x^4-16

   m. $8x^3+27$

6. Simplify a. $\frac{2x^2+7x+3}{2x^2-7x-4}$ b. $\frac{y^2-9}{y^3+27}$ c. $\frac{9x^2-4}{3x^2-5x+2}$

7. Perform the indicated operation.

a. $\frac{x^2-1}{8x-56} \cdot \frac{4x-28}{3x+3}$

b. $\frac{x^2+8x-20}{xy-6y^2} \div \frac{x^2-100y^2}{x^2-16xy+60y^2}$

c. $\frac{5x-35}{x^3+16x} \cdot \frac{2x^3+14x^2}{x^2-49}$

d. $\frac{2x^2-5x-12}{4-x} \div \frac{4x^2-9}{16-x^2}$

e. $\frac{3x}{x^2-4} + \frac{x-2}{x^2-4}$

f. $\frac{x-3}{x+3} - \frac{x}{x-3}$

g. $\frac{2x+5}{x-2} + \frac{5}{x^2+3x-10}$

h. $\frac{2x+3}{x^2+5x+6} - \frac{x-2}{x+3}$

8. Rationalize the denominator.

a. $\frac{3}{\sqrt{5}}$

b. $\frac{2}{2-\sqrt{3}}$

c. $\frac{1+\sqrt{7}}{1-\sqrt{7}}$

d. $\frac{2-3\sqrt{2}}{1+\sqrt{2}}$

9. Solve the following equations.

a. $3(x-5) = 2(x+4)$

b. $\frac{3}{5}(x+3) - 4 = \frac{x}{15}$

c. $(x-2)(x+1) = (x+7)(x-3)$

d. $\frac{3}{x+5} - \frac{x}{x-3} = \frac{7x-x^2}{x^2+2x-15}$

e. $\frac{2-x}{x^2+2x-8} + \frac{3}{x+4} = \frac{1}{x-2}$

f. $\frac{x}{x-4} + \frac{3}{2x+3} = \frac{2x^2+12x-3}{2x^2-5x-12}$

10. Solve the following problems.

(a) A benefit concert for charity was held in the ballroom of a hotel. There were two ways to contribute the charity. Tables in the front of the ballroom could be bought for $2000 each. Individual tickets could be bought for $150 each. A total of 270 tables and individual tickets were bought. The amount raised for charity was $66,400. How many people bought tables?

(b) A store had a promotion on their first day of business to draw in customers. Each customer could choose between a free compilation CD of pop music or a free DVD containing short animated films. The store bought the CDs and the DVDs from a large distribution company for $1.85 and $2.15, respectively. At the end of the day, the store calculated that it had spent a total of $473.25 on the free merchandise. If a total of 243 people received a free CD or DVD, how many DVDs did the store give away?

(c) Harrison and his brother Kelvin are running to the ice cream store. Kelvin can run 8 miles per hour and Harrison can run 5 miles per hour. Kelvin agrees to give his brother a 15 minute head start. How long does it take Kelvin to catch his brother? If the ice cream store is 3.5 miles away, will Kelvin beat his brother to the store?

(d) Bert's car gets 22 mi/gallon city and 39 mi/gallon freeway. This week Bert used 28 gallons of gas and drove 922 miles. How many gallons did Bert use on the freeway?

(e) A chemist wants to create a 100 mL mixture that is 25% salt from a saline solution that is 35% salt and another solution that is 10% salt. How much of the first mixture must she use?

(f) Martha walks or runs 30 miles a week. She runs 6 miles per hour and walks 4 miles per hour. If she spent 5.5 hours running or walking this week, how many miles did she run?

11. Solve by Factoring.

a. $3x^2 + 13x + 14 = 0$
b. $15x^2 - 3x = 8x - 2$
c. $x(x^2 - 2) = 14x$
d. $8(x^2 - x) + 3x = 17(x + 1) - 11$
e. $\frac{(2x-5)(2x+5)}{3} = 5x$

12. Solve by Completing the Square

a. $x^2 + 6x = 5$
b. $x^2 - 10x + 22 = 0$
c. $x^2 - 5 = 3x$
d. $x^2 - 10x + 29 = 0$

13. Solve using the Quadratic Formula.

a. $2x^2 + 5x = 8$
b. $9x^2 + 25 = 30x$
c. $4x^2 = 6x + 5$
d. $3x^2 + 10x + 4 = 2x + 3$
e. $2x^2 - 7x + 11 = 0$
f. $7x^2 + 2 = x$
g. $x^2 - 12x + 61 = 0$

14. Perfrom the following operations.

a. $(2 - 3i) + (4 + i)$
b. $(2 - \sqrt{-25}) + (1 + \sqrt{-4})$
c. $4i - (6 - 3i)$
d. $(1 + 2i)(3 - i)$
e. $(\sqrt{-81})(3 - \sqrt{-17})$
f. $(5 - 4i)(5 + 4i)$
g. $\frac{2-3i}{i}$
h. $\frac{1-\sqrt{-12}}{2+\sqrt{-20}}$
i. $\frac{1+i}{1-i}$
j. $\frac{5+2i}{2-3i}$

15. Solve

a. $|2x + 3| = 9$
b. $2|x - 5| + 2 = 14$
c. $|3x - 1| - 2 = 17$
d. $\sqrt{2x + 3} = x$
e. $2 + \sqrt{x - 2} = x$
f. $\sqrt{2x + 5} = x + 3$
g. $x = 4 + \sqrt{12x - 84}$

16. Solve the following inequalities and leave your answers in interval notation.

a. $3x - 2 \leq 4(x - 6)$
b. $-3 \leq 3 - 2x \leq 14$
c. $\frac{-2}{5}(x - 5) \geq \frac{3}{5}x + 3$
d. $12 \geq 5 - 3x \geq 3$
e. $(x + 2)(x - 3) \geq 0$
f. $x^2 - 3x \geq -2$
g. $\frac{2x-3}{x+5} \geq 0$
h. $\frac{x+3}{x-3} \geq 4$
i. $\frac{(2x+1)(3-5x)}{x-7} \leq 0$
j. $\frac{1-3x}{(x+2)(x-6)} \geq 0$

17. Write the standard form for the equation of the circle with the given center and radius.

a. $(3, 2)$, $r = 4$
b. $(-6, 7)$, $r = \sqrt{3}$
c. $(0, 5)$, $r = 2.5$
d. $(-4.5, -0.7)$, $r = \frac{2}{3}$

18. Graph each cirlce.

a. $(x - 2)^2 + (y - 4)^2 = 25$
b. $x^2 + y^2 + 12x - 2y + 28 = 0$
c. $(x + 3)^2 + y^2 = 36$
d. $x^2 + y^2 + 10x + 8y + 37 = 0$

19. Find the slope of the line passing through the given points.

a. $(-5, 10)$ and $(-8, 12)$
b. $(-3, -8)$ and $(2, -1)$
c. $(5, 6)$ and $(-7, 4)$
d. $(14, -10)$ and $(2, 5)$

20. Find the slope of a line that is perpendicular to the line passing through the given points.

a. $(7, 2)$ and $(5, -3)$
b. $(9, -4)$ and $(1, 2)$
c. $(-3, 4)$ and $(-5, -6)$
d. $(8, 9)$ and $(-5, 10)$

21. Find an equation of the line that passes through the given points.

a. $(4, -3)$ and $(-2, 7)$
b. $(-6, -3)$ and $(2, -1)$
c. $(3, 2)$ and $(-7, 10)$
d. $(6.5, -5)$ and $(7, -3)$
e. $(3, 6)$ and $(3, -4)$
f. $(-2, -7)$ and $(4, -7)$

22. (a) Find an equation of the line that passes through $(8, 7)$ and is perpendicular to $4x + 5y = 2$.

(b) Find an equation of the line that is perpendicular to $3x - 8y = 5$ and passes through the point $(2, 1)$.

(c) Find an equation of the line that is parallel to $5x - 7y = 10$ and passes through the point $(4, -2)$.

(d) Find an equation of the line that passes through $(10, 3)$ and is parallel to $6x + 5y = -2$.

23. Graph the following equations.

a. $2x - 3y = -3$
b. $2x - y = 4$
c. $x + 3y = 6$
d. $x = 3$
e. $5x + 2y = -4$
f. $y = -1$

24. Use the functions f, g, and h defined below to find and simplify each of the following.
$f(x) = 2x^2 - 4x + 1$ $\quad g(t) = \frac{2t+1}{3-t}$ $\quad h(a) = \sqrt{a^2+9}$

a. $f(3)$ b. $g(4)$ c. $h(0)$
d. $g(0)$ e. $h(1)$ f. $f(-4)$
g. $g(-1)$ h. $f(0)$ i. $h(8)$
j. $g(x+2)$ k. $f(3+h)$ ℓ. $\frac{f(3+h)-f(3)}{h}$

25. Let $g(x) = -x^2 + 3x - 6$. Evaluate and simplify each expression.
a. $\dfrac{g(5+h)-g(5)}{h}$ b. $\dfrac{g(a+h)-g(a)}{h}$

26. Find the domain of each function.

a. $f(x) = \frac{x+1}{x+2}$ b. $f(x) = \sqrt{x-7}$ c. $f(x) = \sqrt{6x-2}$
d. $f(x) = \frac{5}{x^2-3x}$ e. $f(x) = \frac{x-5}{\sqrt{x-1}}$ f. $f(x) = \frac{x+2}{\sqrt{x^2-2x-8}}$
g. $f(x) = \sqrt{3x^2+x-2}$ h. $f(x) = \frac{x-3}{x^2+7x-18}$ i. $f(x) = \sqrt{x+3x-4}$

27. If the point $(4, 3)$ is on the graph of f, what is the corresponding point on the graph of $-f(x+2) - 7$.

28. If the point $(-5, 2)$ is on the graph of f, what is the corresponding point on the graph of $3f(-x) + 4$.

29. Use HSRV to graph each function.

a. $f(x) = -(x+4)^2 + 3$ b. $f(x) = \sqrt{1-x} + 3$
c. $f(x) = -\frac{1}{2}(x+2)^3$ d. $f(x) = -2|x-3| + 1$

30. Put the following quadratic function in standard form $f(x) = a(x-k)^2 + h$. State whether each function opens up or down and what the vertex is.

a. $f(x) = x^2 - 16x + 3$ b. $f(x) = 2x^2 - 12x + 4$
c. $f(x) = -3x^2 + 18x - 2$

31. Find the (a) vertex, (b) x-intercepts, (c) y-intercept, (d) maximum or minimum value, and (e) sketch the graph of the following function.

a. $f(x) = -x^2 + 4x - 3$ b. $f(x) = 3x^2 - 12x + 4$

32. Use the functions $f(x) = \sqrt{5x+4}$ and $g(x) = 2x - 3$ to find and simplify the following.

a. $(f \circ g)(x)$ b. $(f+g)(1)$ c. $(f-g)(0)$
d. $(fg)(9)$ e. $\left(\frac{f}{g}\right)(12)$ f. $(g \circ f)(28)$

33. Use the functions $h(x) = \sqrt{x+4}$ and $k(x) = x^2 - 5$ to find and simplify the following.

a. $(k \circ h)(x)$ b. $(h+k)(0)$ c. $(k-h)(12)$
d. $(hk)(2)$ e. $\left(\frac{k}{h}\right)(-3)$ f. $(h \circ k)(\sqrt{2})$

34. Find the inverse of the following functions.

a. $f(x) = 3x - 2$ b. $g(x) = \frac{3}{x-2}$
c. $h(x) = x^2 - 3;\ x \geq 0$ d. $f(x) = (x-2)^2 + 1;\ x \leq 0$

35. Evaluate the following logarithms.

a. $\log_m m$ b. $\log_p p^5$ c. $\ln \sqrt[4]{e}$
d. $\log_\pi \frac{1}{\pi^3}$ e. $\ln \frac{1}{e^8}$ f. $\log_w \sqrt{w}$
g. $\log_4 8$ h. $\log_9 27$ i. $\log_2 16$
j. $\log_8 2$ k. $\log_5(\log_3 3)$ ℓ. $\log_4(\log_9 81)$
m. $\log \frac{1}{100000}$ n. $(\ln e)^7$ o. $\log_7 15$
p. $\log_3 25$ q. $\log_{16} 20$ r. $\log_9 18$

36. Use the properties of logarithms to expand each expression as much as possible.

a. $\ln\left(\frac{a^3b^2}{\sqrt{c}d^5}\right)$ b. $\log_2 \sqrt{\frac{a}{b^3c^5}}$ c. $\log \frac{x^2\sqrt{y}}{1000z}$
d. $\ln \frac{x^3e^5}{10\sqrt{a}}$ e. $\log_3\left(\frac{9\sqrt[3]{x}}{ab^3}\right)$ f. $\log_5\left(\frac{125}{a^2b^3\sqrt[4]{c}}\right)$

37. Write each expression as the logarithm of a single quantity whose coefficient is 1. (Contract)

a. $2\log x + 3\log y - \frac{1}{4}\log z$ b. $\frac{1}{2}\ln a - 3\ln b - 2\ln c$
c. $4\ln x - \frac{1}{3}\ln y + 2\ln w$ d. $5\log_4 a - 3\log_4 b + \log_4 c^2 - \log_4 d$

38. Solve each equation. Approximate solutions to 4 decimal places when necessary.

a. $2^{x+7} = 32$ b. $3^{2x-5} = \frac{1}{81}$ c. $32^{3-2x} = \left(\frac{1}{4}\right)^{2x+9}$
d. $2^{5+3x} = 24$ e. $5^{x+3} = 15$ f. $4e^{2x-3} = 17$

39. Solve each equation.

a. $\ln x + \ln(x-3) = \ln(x+12)$ b. $\log_5(2x+3) - \log_5(x-1) = \log_5 3$
c. $\log_3(x+7) + \log_3(x=1) = \log(10x+22)$ d. $\log(x+11) - \log(x-5) = \log(x+2)$
e. $\log_4(x-7) + \log_4(x+8) = 2$ f. $\log_2(x-6) + \log_2(x+1) = 3$
g. $\log_2(x-4) + \log_2(x-6) = 3$ h. $\log(x-21) + \log x = 2$
i. $\ln(2x+1) + \ln(x-2) = \ln 25$ j. $\log(3x+2) + \log_1 4(x-3) = 1$

40. Fill in the missing parts of the table.

	Amount Invested	**Method of Compounding**	**Annual Interest Rate**	**Accumulated Amount**	**Time (in years)**
a.	\$4000	continuously	2.75%		6
b.	\$6500	monthly	7%		17
c.		continuously	6.5%	\$5000	15
d.		daily	4.25%	\$8800	7
e.	\$700	continuously	1.5%	\$1000	
f.	\$1350	quarterly	12%	\$2000	
g.	\$2400	continuously		\$3500	5
h.	\$1575	weekly		\$2300	3
i.	A_0	continuously	7.25%	double A_0	
j.	A_0	continuously	5.75%	triple A_0	
k.	A_0	continuously		double A_0	4
l.	A_0	continuously		triple A_0	14

41. Solve the following systems using the substitution method.

(a) $\begin{cases} 9x - 3y = 24 \\ 11x + 2y = 1 \end{cases}$

(b) $\begin{cases} 3x - y = 7 \\ 2x + 3y = 1 \end{cases}$

42. Solve the following systems of equations using augmented matrices.

(a) $\begin{cases} x - 2y = 1 \\ 2x - y = 5 \end{cases}$

(b) $\begin{cases} 2x + 4y - 10z = -2 \\ 3x + 9y - 21z = 0 \\ x + 5y - 12z = 1 \end{cases}$

(c) $\begin{cases} 3x + 8y - z = -18 \\ 2x + y + 5z = 8 \\ 2x + 4y + 2z = -4 \end{cases}$

(d) $\begin{cases} 3x - 2y + z = -7 \\ 2x + y - 4z = 0 \\ x + y - 3z = 1 \end{cases}$

(e) $\begin{cases} 3x - 4y - z = 1 \\ 2x - 3y + z = 1 \\ x - 2y + 3z = 2 \end{cases}$

43. Perform the following operations.

(a) $\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -5 \\ 1 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 6 & -2 \\ 1 & -7 \end{bmatrix}$

(c) $5\begin{bmatrix} 1 & -2 & 0 & 4 \\ -3 & 2 & -1 & 6 \end{bmatrix}$

(d) $10\begin{bmatrix} 2 & -1 & 3 \\ 0 & -4 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -6 & 3 \\ 0 & -5 \end{bmatrix}$

(f) $\begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

(g) $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}$

(h) $\begin{bmatrix} -1 & 0 & 2 \\ 4 & -3 & 1 \\ -2 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$

44. Given matrices $A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 4 & 2 \\ 5 & -3 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -5 & 0 \\ 4 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix}$. Let

$$A \cdot B = C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}.$$

Find the following.

(a) c_{23}

(b) c_{31}

(c) c_{31}

(d) The entry in row 2 column 1 of C.

(e) The entry in row 3 column 3 of C.

(f) The entry in row 1 column 2 of C.