Solution HW11 Math 143

Section 7.1

③ $$\frac{\sec^2 2u - 1}{\sec^2 2u} = \sin^2 2u$$

$$\frac{\sec^2 2u - 1}{\sec^2 2u} = \frac{\tan^2 2u}{\sec^2 2u}$$

$$= \frac{\sin^2 2u}{\cos^2 2u} \Big/ \frac{1}{\cos^2 2u} = \frac{\sin^2 2u}{\cos^2 2u} \cdot \frac{\cos^2 2u}{1}$$

$$= \sin^2 2u.$$

⑦ $$\frac{1+\cos 3t}{\sin 3t} + \frac{\sin 3t}{1+\cos 3t} = 2\csc 3t$$

$$\frac{1+\cos 3t}{1+\cos 3t} \cdot \frac{1+\cos 3t}{\sin 3t} + \frac{\sin 3t}{1+\cos 3t} \cdot \frac{\sin 3t}{\sin 3t}$$

$$\frac{1+2\cos 3t+\cos^2 3t}{\sin 3t\,(1+\cos 3t)} + \frac{\sin^2 3t}{\sin 3t\,(1+\cos 3t)}$$

$$\frac{1+2\cos 3t+\cos^2 3t+\sin^2 3t}{\sin 3t\,(1+\cos 3t)}$$

$$\frac{1+2\cos 3t+1}{\sin 3t\,(1+\cos 3t)} = \frac{2+2\cos 3t}{\sin 3t\,(1+\cos 3t)}$$

$$= \frac{2\,\cancel{(1+\cos 3t)}}{\sin 3t\,\cancel{(1+\cos 3t)}}$$

$$= 2\csc 3t.$$

(11) $(\sec u - \tan u)(\csc u + 1) = \cot u$

$$(\sec u - \tan u)(\csc u + 1) = \sec u \csc u - \tan u \csc u + \sec u - \tan u$$

$$= \frac{1}{\cos u}\cdot\frac{1}{\sin u} - \frac{\cancel{\sin u}}{\cos u}\cdot\frac{1}{\cancel{\sin u}} + \frac{1}{\cos u} - \frac{\sin u}{\cos u}$$

$$= \frac{1}{\cos u \sin u} - \cancel{\frac{1}{\cos u}} + \cancel{\frac{1}{\cos u}} - \frac{\sin u}{\cos u}$$

$$= \frac{1}{\cos u \sin u} - \frac{\sin u}{\cos u}\cdot\frac{\sin u}{\sin u}$$

$$= \frac{1}{\cos u \sin u} - \frac{\sin^2 u}{\cos u \sin u}$$

$$= \frac{1 - \sin^2 u}{\cos u \sin u}$$

$$= \frac{\cos^{\cancel{2}} u}{\cancel{\cos u}\sin u}$$

$$= \frac{\cos u}{\sin u} = \cot u$$

(15) $\dfrac{\cos\beta}{1-\sin\beta} = \sec\beta + \tan\beta$

$$\frac{\cos\beta}{1-\sin\beta} = \frac{\cos\beta}{(1-\sin\beta)}\cdot\frac{(1+\sin\beta)}{(1+\sin\beta)}$$

$$= \frac{\cos\beta(1+\sin\beta)}{1-\sin^2\beta}$$

$$= \frac{\cancel{\cos\beta}(1+\sin\beta)}{\cos^{\cancel{2}}\beta}$$

$$= \frac{1+\sin\beta}{\cos\beta} = \frac{1}{\cos\beta} + \frac{\sin\beta}{\cos\beta}$$

$$= \sec\beta + \tan\beta.$$

(21) $\sin^4 r - \cos^4 r = \sin^2 r - \cos^2 r$

$$\sin^4 r - \cos^4 r = (\sin^2 r - \cos^2 r)(\sin^2 r + \cos^2 r)$$
$$= (\sin^2 r - \cos^2 r)\cdot 1$$
$$= \sin^2 r - \cos^2 r$$

(25) $(\sec t + \tan t)^2 = \dfrac{1+\sin t}{1-\sin t}$.

LHS: $(\sec t + \tan t)^2 = \sec^2 t + 2\sec t\tan t + \tan^2 t$.

Now RHS: $\dfrac{1+\sin t}{1-\sin t} = \dfrac{1+\sin t}{1-\sin t}\cdot\dfrac{1+\sin t}{1+\sin t}$

$$= \frac{1+2\sin t+\sin^2 t}{1-\sin^2 t}$$
$$= \frac{1+2\sin t+\sin^2 t}{\cos^2 t}$$
$$= \frac{1}{\cos^2 t} + \frac{2\sin t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t}$$
$$= \sec^2 t + 2\sec t\tan t + \tan^2 t.$$

(35) $\dfrac{\tan\alpha}{1+\sec\alpha} + \dfrac{1+\sec\alpha}{\tan\alpha} = 2\csc\alpha$

$$\frac{\tan\alpha}{1+\sec\alpha} + \frac{1+\sec\alpha}{\tan\alpha} = \frac{\tan\alpha}{\tan\alpha}\cdot\frac{\tan\alpha}{1+\sec\alpha} + \frac{1+\sec\alpha}{\tan\alpha}\cdot\frac{1+\sec\alpha}{1+\sec\alpha}$$
$$= \frac{\tan^2\alpha}{\tan\alpha(1+\sec\alpha)} + \frac{1+2\sec\alpha+\sec^2\alpha}{\tan\alpha(1+\sec\alpha)}$$
$$= \frac{\tan^2\alpha+1+2\sec\alpha+\sec^2\alpha}{\tan\alpha(1+\sec\alpha)}$$
$$= \frac{\sec^2\alpha+2\sec\alpha+\sec^2\alpha}{\tan\alpha(1+\sec\alpha)}$$

contd

$$= \frac{2\sec^2\alpha + 2\sec\alpha}{\tan\alpha\,(1+\sec\alpha)}$$

$$= \frac{2\sec\alpha\,\cancel{(\sec\alpha+1)}}{\tan\alpha\,\cancel{(1+\sec\alpha)}}$$

$$= \frac{2\sec\alpha}{\tan\alpha} = \frac{2}{\cancel{\cos\alpha}}\cdot\frac{\cancel{\cos\alpha}}{\sin\alpha} = 2\csc\alpha$$

(39) $\sec\theta + \csc\theta - \cos\theta - \sin\theta = \sin\theta\tan\theta + \cos\theta\cot\theta$

$$\sec\theta + \csc\theta - \cos\theta - \sin\theta = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} - \cos\theta - \sin\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta\cdot\frac{\cos\theta}{\cos\theta} + \frac{1}{\sin\theta} - \sin\theta\cdot\frac{\sin\theta}{\sin\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} + \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$$

$$= \frac{1-\cos^2\theta}{\cos\theta} + \frac{1-\sin^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$= \sin\theta\tan\theta + \cos\theta\cot\theta.$$

(43) $\dfrac{\cot(-t)+\tan(-t)}{\cot t} = -\sec^2 t$

$$\frac{\cot(-t)+\tan(-t)}{\cot t} = \frac{-\cot t - \tan t}{\cot t}$$

$$= -\frac{\cot t}{\cot t} - \frac{\tan t}{\cot t}$$

$$= -1 - \frac{\tan t}{\frac{1}{\tan t}}$$

$$= -1 - \tan^2 t$$

$$= -1\,(1+\tan^2 t)$$

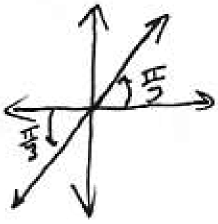
$$= -1\,(\sec^2 t)$$

$$= -\sec^2 t$$

Section 7.2

(3) $\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$



$\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$.

So $\theta = \frac{\pi}{3} + n\pi$ for any integer n.

(7) $\sin x = \frac{\pi}{2}$.

$x = \sin^{-1}(\frac{\pi}{2})$ No Solution $\frac{\pi}{2} > 1$

and $\sin x$ must be between -1 & 1.

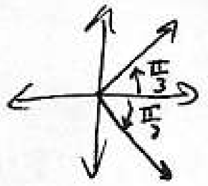
(11) $2\cos 2\theta - \sqrt{3} = 0$

$\cos 2\theta = \frac{\sqrt{3}}{2}$.

Let $x = 2\theta$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$.



$x = \frac{\pi}{3} + 2\pi n$ or $x = \frac{5\pi}{3} + 2\pi n$.

$2\theta = \frac{\pi}{3} + 2\pi n$ or $2\theta = \frac{5\pi}{3} + 2\pi n$

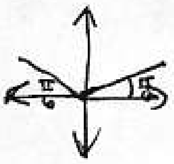
$\theta = \frac{\pi}{6} + \pi n$ or $\theta = \frac{5\pi}{6} + \pi n$. for any integer n.

(15) $\sin(\theta + \frac{\pi}{4}) = \frac{1}{2}$

Let $x = \theta + \frac{\pi}{4}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.



$x = \frac{\pi}{6} + 2\pi n$ or $x = \frac{5\pi}{6} + 2\pi n$

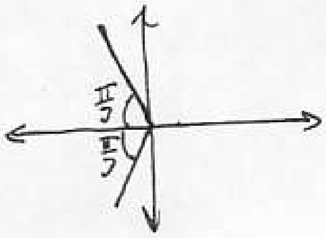
$\theta + \frac{\pi}{4} = \frac{\pi}{6} + 2\pi n$ or $\theta + \frac{\pi}{4} = \frac{5\pi}{6} + 2\pi n$

$\theta = -\frac{\pi}{12} + 2\pi n$ or $\theta = \frac{7\pi}{12} + 2\pi n$ for any integer n.

(19) $2\cos t + 1 = 0$

$\cos t = \frac{-1}{2}$

$t = \frac{2\pi}{3}, \frac{4\pi}{3}$



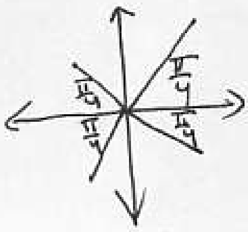
$t = \frac{2\pi}{3} + 2\pi n$ or $\frac{4\pi}{3} + 2\pi n$ for any integer n.

(25) $\sec^2\alpha - 4 = 0$

$\sec^2\alpha = 4$

$\sec\alpha = \pm\sqrt{2}$

$\sec\alpha = \sqrt{2}$ or $\sec\alpha = -\sqrt{2}$



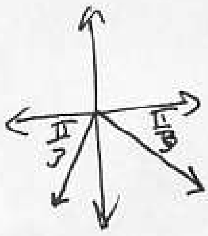
$\alpha = \frac{\pi}{4}$ or $\frac{3\pi}{4}$, or $\frac{5\pi}{4}$, or $\frac{7\pi}{4}$.

$\alpha = \frac{\pi}{4} + n\frac{\pi}{2}$ for any integer n

(27) $\sqrt{3} + 2\sin\beta = 0$

$\sin\beta = \frac{-\sqrt{3}}{2}$

$\beta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$

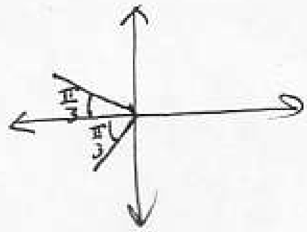


$\beta = \frac{4\pi}{3} + 2\pi n$ or $\frac{5\pi}{3} + 2\pi n$ for any integer n

(19) $2\cos t + 1 = 0$

$\cos t = -\frac{1}{2}$

$t = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$



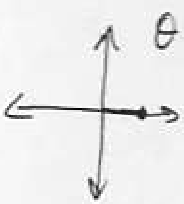
$t = \frac{2\pi}{3} + 2\pi n$ or $\frac{4\pi}{3} + 2\pi n$ for any integer n.

(23) $(\cos\theta - 1)(\sin\theta + 1) = 0$

$\cos\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$\cos\theta = 1$ or $\sin\theta = -1$

$\theta = 0$ $\qquad$ $\theta = \frac{3\pi}{2}$

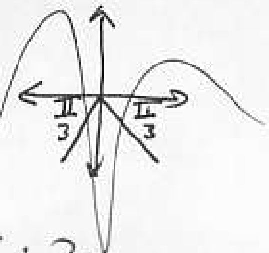


$\theta = 2\pi n$ or $\frac{3\pi}{2} + 2\pi n$ for any integer n.

(27) $\sqrt{3} + 2\sin p = 0$

$\sin\beta = -\frac{\sqrt{3}}{2}$

$\beta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$



$p = \frac{4\pi}{3} + 2\pi n$ or $\frac{5\pi}{3} + 2\pi n$

(37) $\cos\left(2x - \frac{\pi}{4}\right) = 0$ The period of this fctn is π

$2x - \frac{\pi}{4} = \frac{\pi}{2}$ or $2x - \frac{\pi}{4} = \frac{3\pi}{2}$

$2x = \frac{3\pi}{4}$ $\qquad$ $2x = \frac{7\pi}{4}$

$x = \frac{3\pi}{8}$ $\qquad$ $x = \frac{7\pi}{8}$

$x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8} + \pi = \frac{11\pi}{8}, \frac{7\pi}{8} + \pi = \frac{15\pi}{8}.$

$x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}.$