

HW# 13 Solutions

Math 143

Section 7.3

③ a) $\cos\left(\frac{7\pi}{20}\right) = \sin\left(\frac{3\pi}{20}\right)$

b) $\sin\left(\frac{1}{4}\right) = \cos\left(\frac{2\pi-1}{4}\right)$

c) $\tan(1) = \cot\left(\frac{\pi-2}{2}\right)$

d) $\csc(0.53) = \sec\left(\frac{\pi}{2}-0.53\right)$

⑤ a) $\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{3}}{2}$

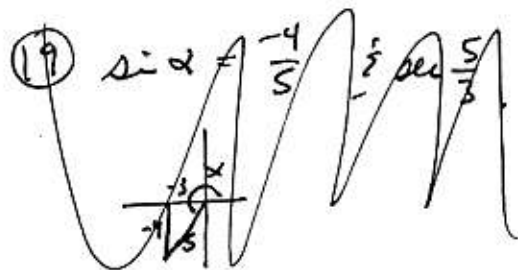
b) ~~$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$~~
 $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$
 $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
 $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

⑦ a) $\tan 60^\circ + \tan 225^\circ$ ~~is~~
 $\sqrt{3} + 1$

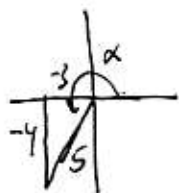
b) $\tan(285^\circ) = \tan(60^\circ + 225^\circ) = \frac{\tan 60^\circ + \tan 225^\circ}{1 - \tan 60^\circ \tan 225^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

⑧ $\cos 48^\circ \cos 23^\circ + \sin 48^\circ \sin 23^\circ$
 $\cos(48^\circ - 23^\circ)$
 $\cos(25^\circ)$

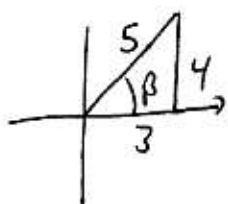
⑨ $\cos 10^\circ \sin 5^\circ - \sin 10^\circ \cos 5^\circ$
 $\sin(5^\circ - 10^\circ)$
 $-\sin 5^\circ$



$$(19) \sin \alpha = \frac{-4}{5} \quad \sec \beta = \frac{5}{3}$$



$$\begin{aligned} \sin \alpha &= \frac{-4}{5} \\ \cos \alpha &= \frac{-3}{5} \\ \tan \alpha &= \frac{4}{3} \end{aligned}$$



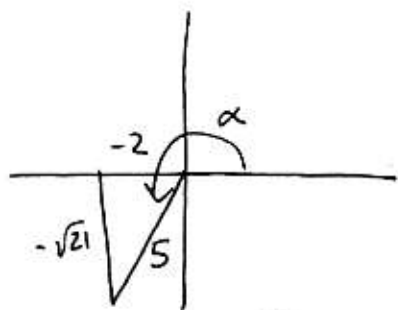
$$\begin{aligned} \sin \beta &= \frac{4}{5} \\ \cos \beta &= \frac{3}{5} \\ \tan \beta &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} (a) \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \left(\frac{-4}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) \\ &= -\frac{12}{25} - \frac{12}{25} = -\frac{24}{25} \end{aligned}$$

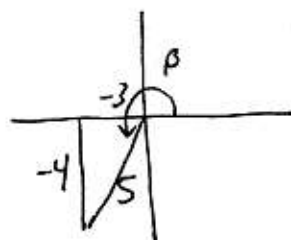
$$\begin{aligned} (b) \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ &= \frac{\frac{4}{3} + \frac{4}{3}}{1 - \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)} = \frac{\frac{8}{3}}{1 - \frac{16}{9}} = \frac{\frac{8}{3}}{\frac{9-16}{9}} = \frac{8}{3} \cdot \frac{9}{-7} = -\frac{24}{7} \end{aligned}$$

(c) $\alpha + \beta$ must be in the 4th quadrant because $\sin(\alpha + \beta) < 0$ and $\tan(\alpha + \beta) < 0$.

$$(21) \alpha \text{ \& } \beta \text{ in III. } \cos \alpha = -\frac{2}{5}, \cos \beta = -\frac{3}{5}$$



$$\begin{aligned} \sin \alpha &= \frac{-\sqrt{21}}{5} \\ \cos \alpha &= \frac{-2}{5} \end{aligned}$$



$$\begin{aligned} \sin \beta &= \frac{-4}{5} \\ \cos \beta &= \frac{-3}{5} \end{aligned}$$

$$\begin{aligned} (a) \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \left(\frac{-\sqrt{21}}{5}\right)\left(\frac{-3}{5}\right) - \left(\frac{-2}{5}\right)\left(\frac{-4}{5}\right) \\ &= \frac{3\sqrt{21} - 8}{25} \end{aligned}$$

$$\begin{aligned} (b) \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \left(\frac{-2}{5}\right)\left(\frac{-3}{5}\right) + \left(\frac{-\sqrt{21}}{5}\right)\left(\frac{-4}{5}\right) \\ &= \frac{6 + 4\sqrt{21}}{25} \end{aligned}$$

(c) $\alpha - \beta$ lies in quadrant I because $\sin(\alpha - \beta) > 0$ & $\cos(\alpha - \beta) > 0$.

$$\begin{aligned}
 (23) \quad \sin(\theta + \pi) &= \sin \theta \cos \pi + \cos \theta \sin \pi \\
 &= \sin \theta (-1) + \cos \theta \cdot (0) \\
 &= -\sin \theta
 \end{aligned}$$

$$\begin{aligned}
 (27) \quad \cos(\theta - \pi) &= \cos \theta \cos \pi + \sin \theta \sin \pi \\
 &= \cos \theta (-1) + \sin \theta (0) \\
 &= -\cos \theta
 \end{aligned}$$

$$(39) \quad \cos(u+v) + \cos(u-v) = 2 \cos u \cos v$$

$$\begin{aligned}
 \cos(u+v) + \cos(u-v) &= \cos u \cos v - \cancel{\sin u \sin v} + \cos u \cos v + \cancel{\sin u \sin v} \\
 &= 2 \cos u \cos v
 \end{aligned}$$

$$(41) \quad \sin(u+v) \sin(u-v) = \sin^2 u - \sin^2 v$$

$$\begin{aligned}
 \sin(u+v) \sin(u-v) &= (\sin u \cos v + \cos u \sin v)(\sin u \cos v - \cos u \sin v) \\
 &= \sin^2 u \cos^2 v - \cos^2 u \sin^2 v \\
 &= \sin^2 u (1 - \sin^2 v) - \cos^2 u (1 - \sin^2 u) \sin^2 v \\
 &= \sin^2 u - \sin^2 u \sin^2 v - \sin^2 v + \sin^2 u \sin^2 v \\
 &= \sin^2 u - \sin^2 v.
 \end{aligned}$$