# HW# 13 Solutions
# Math 143

## Section 7.3

(3) (a) $\cos\left(\frac{7\pi}{20}\right) = \sin\left(\frac{3\pi}{20}\right)$ (b) $\sin\left(\frac{1}{4}\right) = \cos\left(\frac{2\pi-1}{4}\right)$

(c) $\tan(1) = \cot\left(\frac{\pi-2}{2}\right)$ (d) $\csc(0.53) = \sec\left(\frac{\pi}{2}-0.53\right)$

(5) (a) $\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{3}}{2}$.

(b) $\cancel{\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)}$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

(7) (a) $\tan 60^\circ + \tan 225^\circ$

$\sqrt{3} + 1$

(b) $\tan(285^\circ) = \tan(60^\circ + 225^\circ) = \frac{\tan 60^\circ + \tan 225^\circ}{1-\tan 60^\circ \tan 225^\circ} = \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

(11) $\cos 48^\circ \cos 23^\circ + \sin 48^\circ \sin 23^\circ$

$\cos(48^\circ - 23^\circ)$

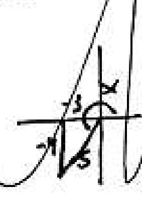
$\cos(25^\circ)$

(13) $\cos 10^\circ \sin 5^\circ - \sin 10^\circ \cos 5^\circ$

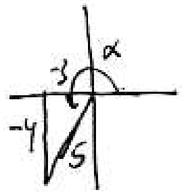
$\sin(5^\circ - 10^\circ)$

$-\sin 5^\circ$.

(19) $\sin\alpha = \frac{-4}{5}$ ; $\sec \frac{5}{3}$



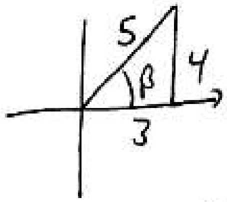
(19) $\sin\alpha = \frac{-4}{5}$ $\quad \sec\beta = \frac{5}{3}$.




$\sin\alpha = \frac{-4}{5}$

$\cos\alpha = -\frac{3}{5}$

$\tan\alpha = \frac{4}{3}$




$\sin\beta = \frac{4}{5}$

$\cos\beta = \frac{3}{5}$

$\tan\beta = \frac{4}{3}$.

(a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{-4}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{-12}{25} - \frac{12}{25} = -\frac{24}{25}$.

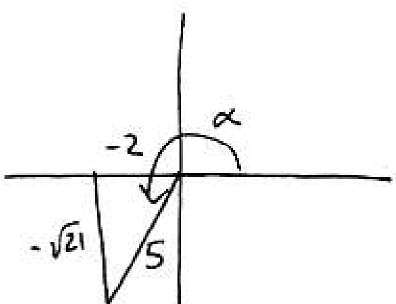
(b) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{4}{3} + \frac{4}{3}}{1 - \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)} \cdot \frac{9}{9}$

$= \frac{12+12}{9-16} = \frac{-24}{7}$.

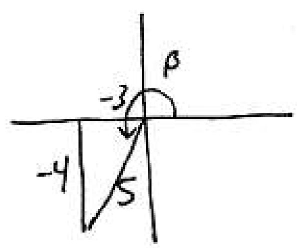
(c) $\alpha+\beta$ must lie in the $4^{th}$ quadrant because $\sin(\alpha+\beta) < 0$ and $\tan(\alpha+\beta) < 0$.

(21) $\alpha$ & $\beta$ in III. $\cos\alpha = -\frac{2}{5}$, $\cos\beta = -\frac{3}{5}$.




$\sin\alpha = \frac{-\sqrt{21}}{5}$

$\cos\alpha = -\frac{2}{5}$




$\sin\beta = -\frac{4}{5}$

$\cos\beta = -\frac{3}{5}$.

(a) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{-\sqrt{21}}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{2}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{3\sqrt{21} - 8}{25}$.

(b) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-2}{5}\right)\left(\frac{-3}{5}\right) + \left(\frac{-\sqrt{21}}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{6 + 4\sqrt{21}}{25}$

(c) $\alpha-\beta$ lies in quadrant I because $\sin(\alpha-\beta) > 0$ & $\cos(\alpha-\beta) > 0$.

(23) $\sin(\theta+\pi) = \sin\theta\cos\pi + \cos\theta\sin\pi$

$= \sin\theta\,(-1) + \cos\theta\cdot(0)$

$= -\sin\theta$

(27) $\cos(\theta-\pi) = \cos\theta\cos\pi + \sin\theta\sin\pi$

$= \cos\theta\,(-1) + \sin\theta\,(0)$

$= -\cos\theta$

(39) $\cos(u+v) + \cos(u-v) = 2\cos u\cos v$

$\cos(u+v) + \cos(u-v) = \cos u\cos v - \cancel{\sin u\sin v} + \cos u\cos v + \cancel{\sin u\sin v}$

$= 2\cos u\cos v$

(41) $\sin(u+v)\sin(u-v) = \sin^2 u - \sin^2 v$

$\sin(u+v)\sin(u-v) = (\sin u\cos v + \cos u\sin v)(\sin u\cos v - \cos u\sin v)$

$= \sin^2 u\cos^2 v - \cos^2 u\sin^2 v$

$= \sin^2 u(1-\sin^2 v) - \cancel{\cos^2 u}\,(1-\sin^2 u)\sin^2 v$

$= \sin^2 u - \sin^2 u\sin^2 v - \sin^2 v + \sin^2 u\sin^2 v$

$= \sin^2 u - \sin^2 v.$