HW 14 Solution
Math 143

Section 7.3

(55) $\sin(4t)\cos t = \sin t \cos(4t)$

$\sin(4t)\cos t - \sin t\cos(4t) = 0$

$\sin(4t)\cos t - \cos(4t)\sin t = 0$

$\sin(4t - t) = 0$

$\sin(3t) = 0$

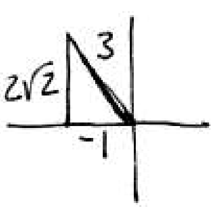
$3t = 0 + \pi n$ for any integer n.

$t = \frac{\pi}{3} n$ for any integer n.

$t = \frac{\pi}{3}$ or $\frac{2\pi}{3}$.

Section 7.4

(3) $\sec\theta = -3$; $90° < \theta < 180°$



$\sin\theta = \frac{2\sqrt{2}}{3}$ $\cos\theta = \frac{-1}{3}$

$\tan\theta = -2\sqrt{2}$.

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right) = -\frac{4\sqrt{2}}{9}$.

$\cos(2\theta) = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{2\sqrt{2}}{3}\right)^2$

$= 1 - 2\cdot\frac{8}{9}$

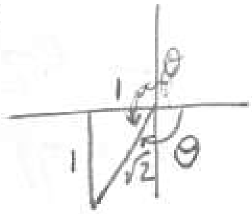
$= -\frac{7}{9}$.

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(-2\sqrt{2})}{1-(-2\sqrt{2})^2} = \frac{-4\sqrt{2}}{-7}$

$= \frac{4\sqrt{2}}{7}$.

⑦ $\tan\theta = 1 \qquad -180° < \theta < -90°$



$\sin\theta = \frac{-\sqrt{2}}{2}, \ \cos\theta = \frac{-\sqrt{2}}{2}.$

Notice
$-90° < \frac{\theta}{2} < -45°$
So $\frac{\theta}{2}$ is in quadrant $\underline{IV}$.

$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{-\sqrt{2+\sqrt{2}}}{2}$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 \cancel{+} \sqrt{2}}}{2}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\cdot\frac{2}{2} = \frac{2+\sqrt{2}}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}+2}{-2} = -\sqrt{2}-1.$$

⑨ ⓐ $\cos(67°30')$

$2\cdot 67°30' = 135°.$

$$\cos(67°30') = \sqrt{\frac{1+\cos(135°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}.$$

ⓑ $\sin(15°)$

$2\cdot 15° = 30°$ So $\sin(15°) = \sqrt{\frac{1-\cos(30°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}\cdot\frac{2}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}.$

ⓒ $\tan\left(\frac{3\pi}{8}\right)$

$2\cdot\frac{3\pi}{8} = \frac{3\pi}{4}$ so $\tan\left(\frac{3\pi}{8}\right) = \frac{1-\cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{3\pi}{4}\right)} = \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}}\cdot\frac{2}{2} = \frac{2+\sqrt{2}}{\sqrt{2}}.$

(13) $4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = 2\sin x$

$$4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = 2\cdot 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$$
$$= 2\sin x$$

(17) $\sin(3u) = \sin u(3 - 4\sin^2 u)$

$$\begin{aligned}
\sin(3u) &= \sin(u + 2u) \\
&= \sin u\cos(2u) + \cos(u)\sin(2u) \\
&= \sin u(1 - 2\sin^2 u) + 2\cos u(2\sin u\cos u) \\
&= \sin u(1 - 2\sin^2 u + 2\cos^2 u) \\
&= \sin u(1 - 2\sin^2 u + 2(1 - \sin^2 u) \\
&= \sin u(1 - 2\sin^2 u + 2 - 2\sin^2 u) \\
&= \sin u(3 - 4\sin^2 u)
\end{aligned}$$

(33) $\sin^4(2x)$

$$\begin{aligned}
&= \left(\sin^2(2x)\right)^2 \\
&= \left(\frac{1 - \cos(4x)}{2}\right)^2 \\
&= \frac{1}{4}\left(1 - 2\cos(4x) + \cos^2(4x)\right) \\
&= \frac{1}{4}\left(1 - 2\cos(4x) + \frac{1 + \cos(8x)}{2}\right) \\
&= \frac{1}{4} - \frac{1}{2}\cos(4x) + \frac{1}{8} + \frac{1}{8}\cos(8x) \\
&= \frac{3}{8} - \frac{1}{2}\cos(4x) + \frac{1}{8}\cos(8x)
\end{aligned}$$

(37) $\cos u + \cos(2u) = 0$

$\cos u + 2\cos^2 u - 1 = 0$

$2\cos^2 u + \cos u - 1 = 0$

$(2\cos u - 1)(\cos u + 1) = 0$

$2\cos u - 1 = 0$ or $\cos u + 1 = 0$

$\cos u = \frac{1}{2}$ $\quad\quad$ $\cos u = -1$

$u = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad\quad$ $u = \pi$.

(47) $\cancel{\cos(3x-3)}$

(a) $\cos(3x) - 3\cos x = 0$

$4\cos^3 x - 3\cos x - 3\cos x = 0$

$4\cos^3 x - 6\cos x = 0$

$2\cos x(2\cos^2 x - 3) = 0$

$2\cos x = 0$ or $2\cos^2 x - 3 = 0$

$\cos x = 0$ or $\cos^2 x = \frac{3}{2}$

$x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \& \frac{3\pi}{2}$ $\quad\quad$ $\cos x = \pm\sqrt{\frac{3}{2}} \approx \pm 1.22$

No Solution

(b) $\sin(3x) - \sin x = 0$

$\sin x(3 - 4\sin^2 x) - \sin x = 0$

$\sin x(3 - 4\sin^2 x - 1) = 0$

$\sin x(2 - 4\sin^2 x) = 0$

$\sin x = 0$ or $2 - 4\sin^2 x = 0$

$\sin^2 x = \frac{1}{2}$

$x = -2\pi, -\pi, 0, \pi, 2\pi$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$