

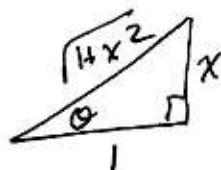
HW 16 Solution Math 143

Section 7.6

(23) $\sin(\tan^{-1}(x))$

$$\tan^{-1}(x) = \theta$$

$$x = \tan \theta$$



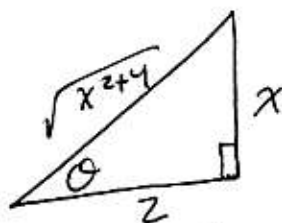
$$\sin(\tan^{-1}(x))$$

$$= \sin \theta = \frac{x}{\sqrt{1+x^2}}$$

(25) $\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right)$

$$\theta = \sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)$$

$$\sin \theta = \frac{x}{\sqrt{x^2+4}}$$



$$\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right) = \sec \theta = \frac{\sqrt{x^2+4}}{2}$$

(55) $\sin^2 x - \sin x - 1 = 0$

Use quadratic formula

$$\sin x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$$

So $\alpha = \pi/2$

$$\sin x = \frac{1+\sqrt{5}}{2}$$

$$x \approx 1.618$$

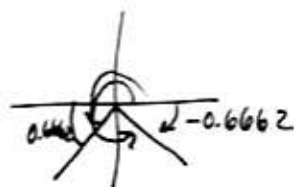
No soln

$$\text{or } \sin x = \frac{1-\sqrt{5}}{2}$$

$$\sin x \approx -0.618$$

$$x = \sin^{-1}(-0.618)$$

$$x \approx -0.6662$$



$$x = 2\pi - 0.6662 \text{ or } \pi + 0.6662$$

$$x \approx 5.6170 \text{ or } 3.8078$$

$$(59) 6\sin^3\theta + 18\sin^2\theta - 5\sin\theta - 15 = 0 \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$6\sin^2\theta(\sin\theta + 3) - 5(\sin\theta + 3) = 0$$

$$(\sin\theta + 3)(6\sin^2\theta - 5) = 0$$

$$\sin\theta + 3 = 0 \quad \text{or} \quad 6\sin^2\theta - 5 = 0$$

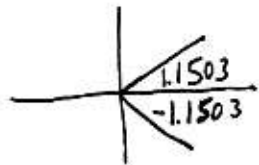
$$\sin\theta = -3 \quad \text{or} \quad \sin^2\theta = \frac{5}{6}$$

No soln

$$\sin\theta = \pm\sqrt{\frac{5}{6}}$$

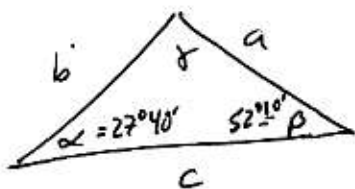
$$\sin\theta \approx 0.9129 \quad \text{or} \quad \sin\theta \approx -0.9129.$$

$$\theta \approx 1.1503 \quad \text{or} \quad \theta \approx -1.1503.$$



Section 8.1

$$(3) \alpha = 27^\circ 40', \beta = 52^\circ 10', a = 32.4$$



$$\gamma = 180^\circ - 27^\circ 40' - 52^\circ 10'$$

$$\gamma = 100^\circ 10'$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin(27^\circ 40')}{32.4} = \frac{\sin(52^\circ 10')}{b}$$

$$\frac{32.4 \sin(52^\circ 10')}{\sin(27^\circ 40')} = b$$

$$55.1 = b$$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\frac{\sin(27^\circ 40')}{32.4} = \frac{\sin(100^\circ 10')}{c}$$

$$c = \frac{32.4 \sin(100^\circ 10')}{\sin(27^\circ 40')}$$

$$c = 68.7$$

Electra 8.7

⑦ $\gamma = 81^\circ, c = 11, b = 12$

$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b}$$

$$\frac{\sin 81^\circ}{11} = \frac{\sin \beta}{12}$$

$$\frac{12 \sin 81^\circ}{11} = \sin \beta$$

$\sin \beta = 1.07$
No Solution.

⑨ $\gamma = 53^\circ 20', a = 140, c = 115$

$$\frac{\sin \gamma}{c} = \frac{\sin \alpha}{a}$$

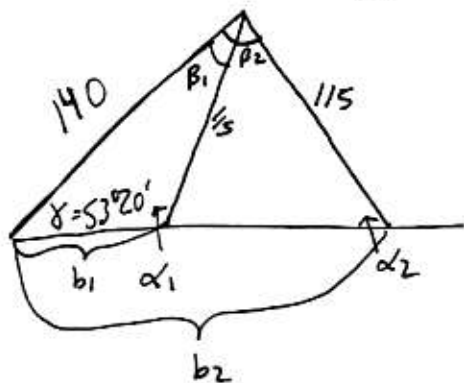
$$\frac{\sin(53^\circ 20')}{115} = \frac{\sin \alpha}{140}$$

$$\frac{140 \sin(53^\circ 20')}{115} = \sin \alpha$$

$$\sin \alpha = 0.9765$$

~~$\alpha \approx 78^\circ$~~ ~~$\alpha = 180^\circ - 78^\circ = 102^\circ$~~

$\alpha \approx 78^\circ 30'$ or $\alpha = 180^\circ - 78^\circ 30' = 101^\circ 30'$



Since $101^\circ 30' + 53^\circ 20' < 180^\circ$
we have two triangles.

$$\alpha_1 = 101^\circ 30'$$

$$\alpha_2 = 78^\circ 30'$$

$$\beta_1 = 180^\circ - 101^\circ 30' - 53^\circ 20' = 25^\circ 10'$$

$$\beta_2 = 180^\circ - 78^\circ 30' - 53^\circ 20' = 48^\circ 10'$$

$$\frac{\sin \beta_1}{b_1} = \frac{\sin \gamma}{c}$$

$$b_1 = \frac{115 \sin(25^\circ 10')}{\sin(53^\circ 20')} \approx 61$$

$$\frac{\sin \beta_2}{b_2} = \frac{\sin \gamma}{c}$$

$$b_2 = \frac{115 \sin(48^\circ 10')}{\sin(53^\circ 20')} \approx 107$$

$$\textcircled{13} \quad \alpha = 65^\circ 10' \quad a = 21.3 \quad b = 18.9$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

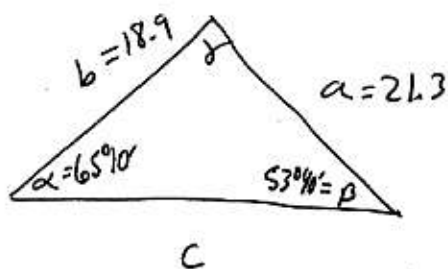
$$\frac{\sin(65^\circ 10')}{21.3} = \frac{\sin \beta}{18.9}$$

$$0.8053 \approx \sin \beta$$

$$53^\circ 40' \approx \beta$$

$$\text{or } \beta = 180^\circ - 53^\circ 40' \\ = 126^\circ 20'$$

Since $126^\circ 20' + 65^\circ 10' > 180^\circ$, there is only one triangle.



$$\gamma = 180^\circ - 65^\circ 10' - 53^\circ 40' \\ \gamma = 61^\circ 10'$$

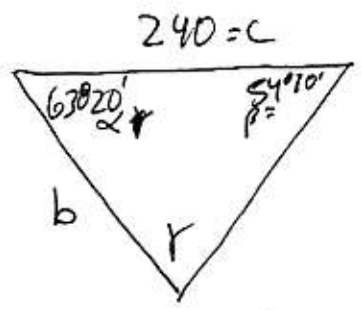
$$\frac{\sin \gamma}{c} = \frac{\sin \alpha}{a}$$

$$\frac{\sin(61^\circ 10')}{c} = \frac{\sin(65^\circ 10')}{21.3}$$

$$c = \frac{21.3 \sin(61^\circ 10')}{\sin(65^\circ 10')}$$

$$c \approx 20.6$$

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$$\gamma = 180^\circ - 63^\circ 20' - 54^\circ 10'$$

$$\gamma = 62^\circ 30'$$

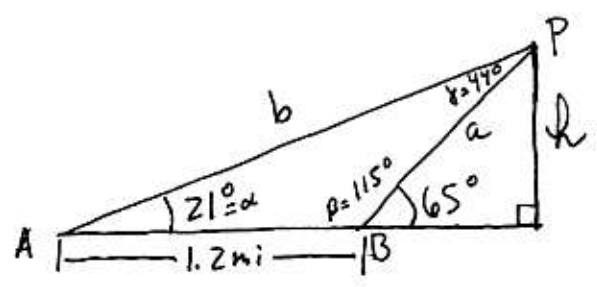
$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b}$$

$$\frac{\sin(62^\circ 30')}{240} = \frac{\sin(54^\circ 10')}{b}$$

$$b = \frac{240 \sin(54^\circ 10')}{\sin(62^\circ 30')}$$

$$b = 219 \text{ ft.}$$

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$$\textcircled{a} \quad \frac{\sin 115^\circ}{b} = \frac{\sin 44^\circ}{1.2}$$

$$\frac{1.2 \sin 115^\circ}{\sin 44^\circ} = b$$

$$1.6 \text{ miles} \approx b.$$

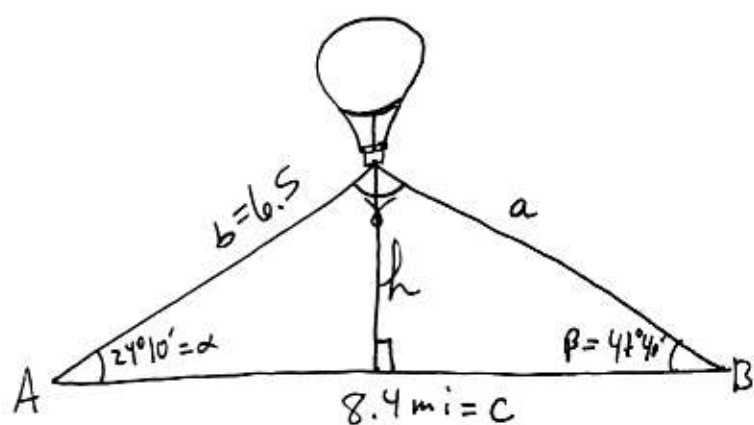
$$\textcircled{b} \quad \sin \alpha = \frac{h}{b}$$

$$\sin 21^\circ = \frac{h}{1.6}$$

$$1.6 \sin 21^\circ = h$$

$$0.6 \text{ miles} = h$$

(21)



$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b}$$

$$\frac{\sin \gamma}{8.4} = \frac{\sin 47^\circ 40'}{b}$$

$$\frac{\sin(108^\circ 10')}{8.4} = \frac{\sin(47^\circ 40')}{b}$$

$$b = \frac{8.4 \sin(47^\circ 40')}{\sin(108^\circ 10')}$$

$$b = 6.5$$

$$\sin \alpha = \frac{h}{b}$$

$$\sin(24^\circ 10') = \frac{h}{6.5}$$

$$6.5 \sin(24^\circ 10') = h$$

$$2.7 = h$$

2.7 miles

$$\gamma = 180^\circ - 24^\circ 10' - 47^\circ 40'$$

$$\gamma = 108^\circ 10'$$