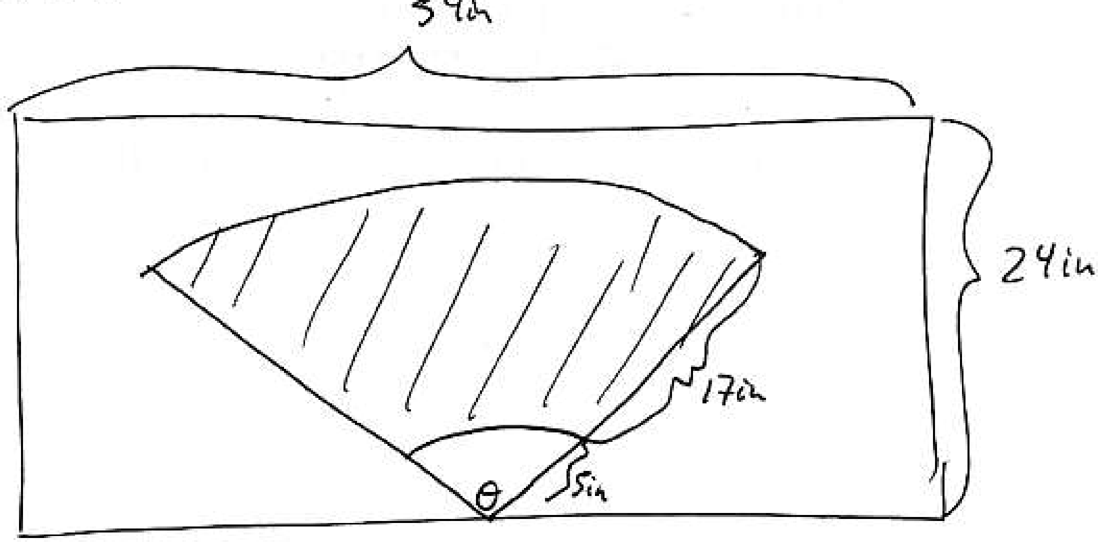
HW 5-20-09

Math 143

Section 6.1

(41)



$\theta = 120° = \frac{2\pi}{3}$

Area of the window $= 24 \cdot 54 = 1296 in^2$

Area of the large sector $= \frac{1}{2}(22)^2 \frac{2\pi}{3} \approx 506.8\ in^2$

Area of the small sector $= \frac{1}{2}(5)^2 \frac{2\pi}{3} \approx 26.2\ in^2$.

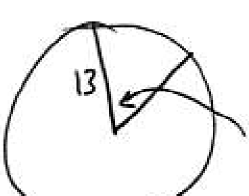
The area of the wiped region is $506.8 - 26.2 = 480.6 in^2$.

Percentage of area $\frac{480.6}{1296} \approx 37\%$

(51)

$\theta = \frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$

Area of a slice $\frac{1}{2}(9)^2 \cdot \frac{\pi}{3} = \frac{27\pi}{2} \approx 42.4\ in^2$



$\theta = \frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$

Area of a slice

$\frac{1}{2}(13)^2 \cdot \frac{\pi}{4} = \frac{169\pi}{8} \approx 66.4 in^2$.

So the small slice is $\frac{42.4}{2} = 21.2 \text{ in}^2/\text{dollar}$

The large slice is $\frac{66.4}{3} \approx 22.1 \text{ in}^2/\text{dollar}$.

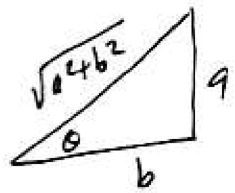
The large slice is a better deal, but not by much

Section 6.2

(3) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

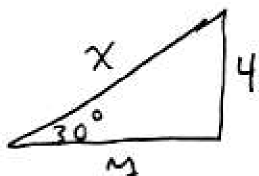
(7)



$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$ $\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$ $\tan\theta = \frac{a}{b}$

$\csc\theta = \frac{\sqrt{a^2+b^2}}{a}$ $\sec\theta = \frac{\sqrt{a^2+b^2}}{b}$ $\cot\theta = \frac{b}{a}$.

(11)



$\tan 30° = \frac{4}{y}$

$\frac{\sqrt{3}}{3} = \frac{4}{y}$

$y\sqrt{3} = 12$

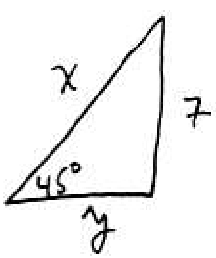
$y = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

$\sin 30° = \frac{4}{x}$

$\frac{1}{2} = \frac{4}{x}$

$x = 8$

(13)



$\tan 45° = \frac{7}{y}$

$1 = \frac{7}{y}$

$y = 7$

$\sin 45° = \frac{7}{x}$

$\frac{\sqrt{2}}{2} = \frac{7}{x}$

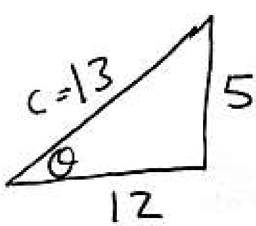
$x\sqrt{2} = 14$

$x = \frac{14}{\sqrt{2}} = 7\sqrt{2}$.

(19) $\tan\theta = \frac{5}{12}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$.

So we draw a triangle representing this.



$c^2 = 5^2 + 12^2$

$c^2 = 25 + 144$

$c^2 = 169$

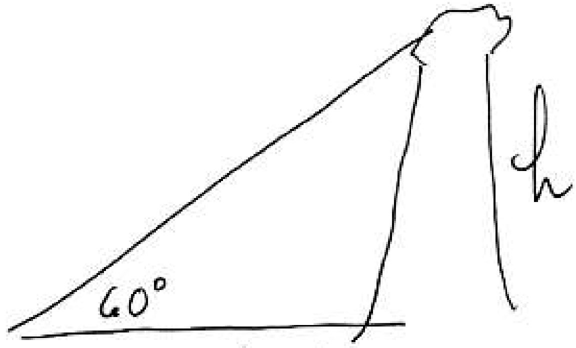
$c = 13$

$\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$

$\csc\theta = \frac{13}{5}$ $\sec\theta = \frac{13}{12}$

$\cot\theta = \frac{12}{5}$

(23)



200 ft

$$\tan 60° = \frac{h}{200}$$

$$\sqrt{3} = \frac{h}{200}$$

$$200\sqrt{3} = h$$

(29) (a) 0.669 (b) 0.2250 (c) $\csc 123° = \frac{1}{\sin 123°} = \frac{1}{0.8387} = 1.1924$

(d) $\sec(-190°) = \frac{1}{\cos(-190°)} = -1.0154$

(31) (a) $\cot \frac{\pi}{13} = \frac{1}{\tan(\frac{\pi}{13})} = 4.0572$ (b) $\csc 1.32 = \frac{1}{\sin 1.32} \approx 1.0323$

(c) $\cos(-8.54) = -0.6335$ (d) $\tan\left(\frac{3\pi}{7}\right) \approx 4.3813$

(35) (a) $\tan^2 4\beta - \sec^2 4\beta$

Recall that $\tan^2\theta + 1 = \sec^2\theta$

So $\tan^2\theta - \sec^2\theta = -1$

So $\tan^2 4\beta - \sec^2 4\beta = -1$

(b) $4\tan^2\beta - 4\sec^2\beta$

$= 4(\tan^2\beta - \sec^2\beta)$

$= -4.$

(43) Skip

(49) $\cos\theta \cdot \sec\theta = 1$

$$\cos\theta \cdot \sec\theta = \cos\theta \cdot \frac{1}{\cos\theta} = 1$$

(53) $\frac{\csc\theta}{\sec\theta} = \cot\theta$

$$\frac{\csc\theta}{\sec\theta} = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{1}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

(57) $\cos^2\theta(\sec^2\theta - 1) = \sin^2\theta$

Note: $\tan^2\theta + 1 = \sec^2\theta$

so $\tan^2\theta = \sec^2\theta - 1$

$\cancel{\cos^2\theta(\tan^2\theta)}$

$$\cos^2\theta(\sec^2\theta - 1) = \cos^2\theta\cdot\tan^2\theta$$

$$= \cancel{\cos^2\theta}\cdot\frac{\sin^2\theta}{\cancel{\cos^2\theta}}$$

$$= \sin^2\theta.$$

(63) $\sec\theta - \cos\theta = \tan\theta\sin\theta$

$$\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}$$ ← Common denominator is $\cos\theta$

$$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\sin\theta$$

$$= \tan\theta\sin\theta$$

(65) $(\cot\theta + \csc\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$

Multiply these out →

$$(\cot\theta + \csc\theta)(\tan\theta - \sin\theta) = \cot\theta\tan\theta - \cot\theta\sin\theta + \csc\theta\tan\theta - \csc\theta\sin\theta$$

$$= \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} - \frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1} + \frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta} - \frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1}$$

$$1 - \cos\theta + \frac{1}{\cos\theta} - 1$$

$$\sec\theta - \cos\theta$$

67 $\sec^2 3\theta \csc^2 3\theta = \sec^2 3\theta + \csc^2 3\theta$

$$\begin{aligned}
\sec^2 3\theta \csc^2 3\theta &= (\tan^2 3\theta + 1)\csc^2 3\theta \\
&= \tan^2 3\theta \csc^2 3\theta + \csc^2 3\theta \\
&= \frac{\cancel{\sin^2 3\theta}}{\cos^2 3\theta} \cdot \frac{1}{\cancel{\sin^2 3\theta}} + \csc^2 3\theta \\
&= \frac{1}{\cos^2 3\theta} + \csc^2 3\theta \\
&= \sec^2 3\theta + \csc^2 3\theta
\end{aligned}$$