

## MATH 229 Review Sheet for Exam 2

### 1. Section 10.6 The Derivative

(a) Use the definition of the derivative  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to find the derivative of the following functions.

i.  $f(x) = 3$

ii.  $f(x) = x^2 - 3x + 1$

iii.  $f(x) = \sqrt{x}$

iv.  $f(x) = -3x^2 + 4$

### 2. Sections 11.1-11.3 Rules of Differentiation

(a) Find the first derivative of the following functions.

i.  $f(x) = -2x^3$

ii.  $f(x) = .002x^3 - .1x^2 + .07x - 3$

iii.  $f(x) = 15x^{3/5}$

iv.  $f(x) = -3\sqrt[3]{x}$

v.  $f(t) = \frac{5}{t^3} - \frac{3}{t^2} + \frac{1}{t}$

vi.  $f(x) = (x^3 - 13x + 5)(x^2 + 1)$

vii.  $f(x) = (2x^2 + 3)\sqrt{x}$

viii.  $f(t) = (1 + \sqrt{t})(t^3 + 4t)$

ix.  $f(x) = \frac{1}{x^2+1}$

x.  $f(u) = \frac{u}{1-u^2}$

xi.  $f(x) = \frac{x^2+2}{x^2+3x-3}$

xii.  $f(x) = \frac{x^3-2}{x^2+1}$

xiii.  $f(t) = 2(t^4 - t)^5$

xiv.  $f(t) = \frac{1}{t^3-1}$

xv.  $f(x) = \frac{1}{\sqrt{3x^3-1}}$

xvi.  $f(t) = \frac{(2t-1)^2}{t+1}$

xvii.  $f(x) = \frac{2x^3}{(x^2-1)^3}$

xviii.  $f(t) = (t^{-1} - t^{-2})^3$

xix.  $g(t) = \frac{1}{2}(2t^2 + t)^{-3}$

(b) Find the slope and the equation of the tangent line to the previous functions at the following points.

i.  $(-1, 2)$

ii. Skip

iii.  $(32, 120)$

- iv.  $(1, -3)$
- v.  $(1, 3)$
- vi.  $(0, 5)$
- vii.  $(1/4, 25/16)$
- viii.  $(1, 10)$
- ix.  $(3, 1/10)$
- x.  $(0, 0)$
- xi.  $(1, 3)$
- xii.  $(2, 6/5)$
- xiii.  $(0, 0)$
- xiv.  $(-1, -1/2)$
- xv. Skip
- xvi.  $(1, 1/2)$

(c) Find the points where the tangent line to the given function has the given slope  $m$ .

- i.  $f(x) = x^2 - 3x + 4$ ;  $m = 3$
- ii.  $f(x) = x^3 - 2x^2 - 2$ ;  $m = 0$
- iii.  $f(x) = \frac{-2}{x}$ ;  $m = 8$
- iv.  $f(x) = \frac{1}{3}x^3 - 3x^2 + 8x - 10$ ;  $m = 0$
- v. Just find the values of  $x$  where,  $f(x) = x^3 - 3x^2 + 2x - 1$  has slope  $m = 0$ .
- vi.  $f(x) = \frac{1}{x^2}$ ;  $m = 1/4$

(d) Find  $\frac{dy}{du}$ ,  $\frac{du}{dx}$  and  $\frac{dy}{dx}$ .

- i.  $y = u^3$  and  $u = 2x^2 + 3x$
- ii.  $y = u^{3/2}$  and  $u = x^3 + 4$
- iii.  $y = \sqrt{u} + \frac{1}{\sqrt{u}}$  and  $u = x^2 + 2x$

(e) Let  $h(x) = f(g(x))$  and let  $f'(2) = -4$ ,  $f(3) = 2$ ,  $g(5) = 2$  and  $g'(5) = -1$ , find  $h'(5)$ .

(f) Let  $k(x) = g(f(x))$  and let  $f'(1) = 3$ ,  $f(1) = -7$ ,  $g(3) = 14$  and  $g'(-7) = 1/9$ , find  $k'(1)$ .

### 3. Section 11.4 Marginal Economic Functions

(a) page 691 5-13 odd (You needn't do any of the graphing.)

### 4. Section 11.5 Higher Order Derivatives

(a) Find the derivatives of all orders of the following functions.

- i.  $f(x) = x^5 - 3x^2 + 2x + 1$
- ii.  $g(x) = x^6 - x^5 + x^4 + x^3 - x^2 - x + 1$

(b) Find the first and second derivatives of the following functions.

i.  $f(x) = \frac{2x}{x^2-1}$

ii.  $g(x) = \sqrt{x-3}$

iii.  $f(x) = (x^2 - 3)^2(x + 2)$

iv.  $f(x) = \frac{x-1}{x+1}$

v.  $f(x) = 3x^{-3} - 4x^{-2} + x^{-1} - 2$

vi. Find the third derivative as well of  $f(x) = \frac{1}{x}$

(c) page 699 #31

(d) page 699 #35

## 5. Sections 12.1 and 12.2 Applications of the First and Second Derivatives

(a) Find the intervals where the following functions are increasing and decreasing.

i.  $f(x) = 8x^2 - 12x + 10$

ii.  $g(x) = \frac{1}{3}x^3 - 3x^2 - 16x + 3$

iii.  $f(x) = x^4 - \frac{9}{2}x^2 - 8$

iv.  $g(x) = x^3 - 2x^2 + 2x - 2$  Hint: Use the quadratic formula.

v.  $h(x) = 1 - \frac{1}{x}$

vi.  $f(x) = \frac{x^2+1}{x}$

vii.  $g(x) = x\sqrt{x}$

viii.  $h(x) = x^{2/3}(x^2 - 1)$

(b) Find the critical points of the functions above and determine whether they are relative maximums, relative minimums, or neither.

(c) Determine the intervals on which the functions above are concave up and concave down.

(d) Determine the inflection points of the functions above.

(e) Use the second derivative test to find the relative extrema of the following functions.

i.  $f(x) = x^3 - \frac{1}{2}x^2 - 2x + 1$

ii.  $g(x) = x^4 + 2x^3 + x^2 - 15$

iii.  $h(x) = \frac{1}{x^4+1}$

iv.  $f(x) = x^3 + 4x^2 - 3x - 10$

## 6. Section 12.3 Graphing Functions

(a) Determine the vertical and horizontal asymptotes of the following functions.

i.  $f(x) = x^4 - \frac{9}{2}x^2 - 8$

ii.  $g(x) = x^3 - 2x^2 + 2x - 2$

- iii.  $h(x) = 1 - \frac{1}{x}$
- iv.  $f(x) = \frac{x^2+1}{x}$
- v.  $g(x) = x\sqrt{x}$
- vi.  $h(x) = x^{2/3}(x^2 - 1)$

(b) Graph the following functions using the information you determine above.

- i.  $f(x) = 8x^2 - 12x + 10$
- ii.  $g(x) = \frac{1}{3}x^3 - 3x^2 - 16x + 3$
- iii.  $f(x) = x^4 - \frac{9}{2}x^2 - 8$
- iv.  $g(x) = x^3 - 2x^2 + 2x - 2$
- v.  $h(x) = 1 - \frac{1}{x}$
- vi.  $f(x) = \frac{x^2+1}{x}$
- vii.  $g(x) = x\sqrt{x}$
- viii.  $h(x) = x^{2/3}(x^2 - 1)$

## 7. Section 12.4 and 12.5 Optimization I and II

(a) Find the absolute maximum and minimum of the following functions on the given intervals.

- i.  $f(x) = x^3 + 3x^2 - 1; [-3, 2]$
- ii.  $f(x) = 3x^4 + 4x^3; [-2, 1]$
- iii.  $f(x) = \frac{x+1}{x-1}; [2, 4]$
- iv.  $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}; [0, 3]$

(b) page 791 problems 41,45,47

(c) page 792 problem 53

(d) page 803-804 problems 3,5,9,21