MATH 229 Review Sheet for Exam 2

1. Section 10.6 The Derivative

   (a) Use the definition of the derivative $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to find the derivative of the following functions.

      i. $f'(x) = 0$
      ii. $f'(x) = 2x - 3$
      iii. $f'(x) = \frac{1}{2\sqrt{x}}$
      iv. $f'(x) = -6x$

2. Sections 11.1-11.3 Rules of Differentiation

   (a) Find the first derivative of the following functions.

      i. $f'(x) = -6x^2$
      ii. $f'(x) = .006x^2 - .2x + .07$
      iii. $f'(x) = \frac{3}{\sqrt[5]{x^2}}$
      iv. $f'(x) = \frac{-1}{\sqrt[3]{x^2}}$
      v. $f'(t) = \frac{-15}{t^4} + \frac{6}{t^3} - \frac{1}{t^2}$
      vi. $f'(x) = 3x^4 + 6x^3 - 36x^2 + 10x - 13$
      vii. $f'(x) = \frac{10x^2+3}{2\sqrt{x}}$
      viii. $f'(t) = \frac{7t^3+12t+6t^2\sqrt{t}+8\sqrt{t}}{2\sqrt{t}}$
      ix. $f'(x) = \frac{-2x}{(x^2+1)^2}$
      x. $f'(u) = \frac{3u^3}{(1-u^2)^2}$
      xi. $f'(x) = \frac{3x^2-10x-6}{(x^2+3x-3)^2}$
      xii. $f'(x) = \frac{x^4+3x^2-4x}{(x^2+1)^2}$
      xiii. $f'(t) = 10(t^4 - t)^4(4t^3 - 1)$
      xiv. $f'(t) = \frac{-3t^2}{(t^3-1)^2}$
      xv. $f'(x) = \frac{-9x^2}{2\sqrt{(3x^3-1)^3}}$
      xvi. $f'(t) = \frac{4t^2+8t-5}{t+1}$
      xvii. $f'(x) = \frac{-6x^2}{(x^2-1)^4}$
      xviii. $f'(t) = 3(t^{-1} - t^{-2})^2(-t^{-2} + 2t^{-3})$
      xix. $g'(t) = \frac{-12t-3}{2(2t^2+t)^4}$

   (b) Find the slope and the equation of the tangent line to the previous functions at the following points.

      i. Slope:-6, Equation of Tangent Line:$y = -6x - 4$

ii. Skip
iii. Slope:$\frac{3}{4}$, Equation of Tangent Line:$y = \frac{3}{4}x + 28$
iv. Slope:-1, Equation of Tangent Line:$y = -x - 2$
v. Slope:-10, Equation of Tangent Line:$y = -10x + 13$
vi. Slope:-13, Equation of Tangent Line:$y = -13x + 5$
vii. Slope:$\frac{29}{8}$, Equation of Tangent Line:$y = \frac{29}{8}x + \frac{21}{32}$
viii. Slope:$\frac{33}{2}$, Equation of Tangent Line:$y = \frac{33}{2}x - \frac{13}{2}$
ix. Slope:$\frac{-3}{50}$, Equation of Tangent Line:$y = \frac{-3}{50}x + \frac{7}{25}$
x. Slope:0, Equation of Tangent Line:$y = 0$
xi. Slope:-13, Equation of Tangent Line:$y = -13x + 16$
xii. Slope:$\frac{4}{5}$, Equation of Tangent Line:$y = \frac{4}{5}x - \frac{2}{5}$
xiii. Slope:0, Equation of Tangent Line:$y = 0$
xiv. Slope:$\frac{-3}{4}$, Equation of Tangent Line:$y = \frac{-3}{4}x - \frac{5}{4}$
xv. Skip
xvi. Slope:$\frac{7}{2}$, Equation of Tangent Line:$y = \frac{7}{2}x - 3$

(c) Find the points where the tangent line to the given function has the given slope m.

i. $(3, 7)$
ii. $(0, -2)$ and $(\frac{4}{3}, \frac{-86}{27})$
iii. $(\frac{1}{2}, -4)$, $(\frac{-1}{2}, 4)$
iv. $(4, \frac{-14}{3})$, $(2, \frac{-10}{3})$
v. $\frac{3 \pm \sqrt{3}}{3}$
vi. $(-1, 1)$

(d) Find $\frac{dy}{du}$, $\frac{du}{dx}$ and $\frac{dy}{dx}$.

i. $\frac{dy}{du} = 3u^2$, $\frac{du}{dx} = 4x + 3$, $\frac{dy}{dx} = 3(2x^2 + 3x)^2(4x + 3)$
ii. $\frac{dy}{du} = \frac{3}{2}\sqrt{u}$, $\frac{du}{dx} = 3x^2$, $\frac{dy}{dx} = \frac{9x^2}{2}\sqrt{x^3 + 4}$
iii. $\frac{dy}{du} = \frac{1}{2\sqrt{u}} - \frac{1}{2\sqrt{u^3}}$, $\frac{du}{dx} = 2x + 2$, $\frac{dy}{dx} = \frac{2x+2}{2\sqrt{x^2+2x}} - \frac{2x+2}{2\sqrt{(x^2+2x)^3}}$

(e) $h'(5) = 4$.

(f) $k'(1) = \frac{1}{3}$.

3. Section 11.4 Marginal Economic Functions

(a) page 691 5-13 odd (Answers are in the back of your book.)

4. Section 11.5 Higher Order Derivatives

(a) Find the derivatives of all orders of the following functions.

i. $f'(x) = 5x^4 - 6x + 2$
$f''(x) = 20x^3 - 6$
$f'''(x) = 60x^2$
$f^{(4)}(x) = 120x$
$f^{(5)}(x) = 120$
$f^{(n)}(x) = 0$ for $n \geq 6$

ii. $g'(x) = 6x^5 - 5x^4 + 4x^3 + 3x^2 - 2x - 1$
$g''(x) = 30x^4 - 20x^3 + 12x^2 + 6x - 2$
$g'''(x) = 120x^3 - 60x^2 + 24x + 6$
$g^{(4)}(x) = 360x^2 - 120x + 24$
$g^{(5)}(x) = 720x - 120$
$g^{(6)}(x) = 720$
$g^{(n)}(x) = 0$ for $n \geq 7$

(b) Find the first and second derivatives of the following functions.

i. $f'(x) = \frac{-2x^2-2}{(x^2-1)^2}$
$f''(x) = \frac{4x^3+12x}{(x^2-1)^3}$

ii. $g'(x) = \frac{1}{2\sqrt{x-3}}$
$f''(x) = \frac{-1}{4\sqrt{(x-3)^3}}$

iii. $f'(x) = (x^2 - 3)(5x^2 + 8x - 3)$
$f''(x) = 20x^3 + 24x^2 - 36x - 24$

iv. $f'(x) = \frac{2}{(x+1)^2}$
$f''(x) = \frac{-4}{(x+1)^3}$

v. $f'(x) = -9x^{-4} + 8x^{-3} - x^{-2}$
$f''(x) = 36x^{-5} - 24x^{-4} + 2x^{-3}$

vi. $f'(x) = \frac{-1}{x^2}$
$f''(x) = \frac{2}{x^3}$
$f'''(x) = \frac{-6}{x^4}$

(c) page 699 #31 (Answers in the back of your book.)

(d) page 699 #35 (Answers in the back of your book.)

5. Sections 12.1 and 12.2 Applications of the First and Second Derivatives

(a) Find the intervals where the following functions are increasing and decreasing.

i. Decreasing on $(-\infty, \frac{3}{4})$ and increasing on $(\frac{3}{4}, \infty)$.

ii. Decreasing on $(-2, 8)$ and increasing on $(-\infty, -2)$ and $(8, \infty)$.

iii. Decreasing on $(-\infty, \frac{-3}{2})$ and $(0, \frac{3}{2})$ and increasing on $(\frac{-3}{2}, 0)$ and $(\frac{3}{2}, \infty)$

iv. Increasing on $(-\infty, \infty)$

v. Increasing on $(-\infty, 0)$ and $(0, \infty)$

vi. Decreasing on $(-1, 0)$ and $(0, 1)$ and increasing on $(-\infty, -1)$ and $(1, \infty)$

vii. The domain of $g(x)$ is $[0,\infty]$ and is increasing on $(0,\infty)$.

viii. Decreasing on $(-\infty,\frac{-1}{2})$ and $(0,\frac{1}{2})$ and increasing on $(\frac{-1}{2},0)$ and $(\frac{1}{2},\infty)$

(b) Find the critical points of the functions above and determine whether they are relative maximums, relative minimums, or neither.

i. Critical point is $(\frac{3}{4},\frac{11}{2})$ and it's a minimum.

ii. Critical points are $(-2,\frac{61}{3})$, which is a maximum, and $(8,\frac{439}{3})$, which is a minimum.

iii. Critical points are $(\frac{-3}{2},\frac{-209}{16})$, which is a minimum, $(0,-8)$, which is a maximum, and $(\frac{3}{2},\frac{-209}{16})$, which is a minimum.

iv. No critical points.

v. No critical points because 0 is not in the domain of $h(x)$.

vi. Critical points are $(-1,-2)$, which is a maximum, and $(1,2)$, which is a minimum.

vii. No critical points.

viii. Critical points $(\frac{-1}{2},\frac{-3\sqrt[3]{2}}{8})$, which is a minimum, $(0,0)$, which is a maximum, and $(\frac{1}{2},\frac{-3\sqrt[3]{2}}{8})$, which is a minimum.

(c) Determine the intervals on which the functions above are concave up and concave down.

i. Concave up on $(-\infty,\infty)$.

ii. Concave down on $(-\infty,3)$ and concave up on $(3,\infty)$.

iii. Concave up on $(-\infty,\frac{-\sqrt{3}}{2})$ and $(\frac{\sqrt{3}}{2},\infty)$ and concave down on $(\frac{-\sqrt{3}}{2},\frac{\sqrt{3}}{2})$

iv. Concave down on $(-\infty,\frac{2}{3})$ and concave up on $(\frac{2}{3},\infty)$

v. Concave up on $(-\infty,0)$ and concave down on $(0,\infty)$

vi. Concave down on $(-\infty,0)$ and concave up on $(0,\infty)$

vii. Concave up on $(0,\infty)$

viii. Concave up on $(-\infty,0)$ and $(0,\infty)$

(d) Determine the inflection points of the functions above.

i. No inflection points

ii. $(3,46)$

iii. $(\frac{-\sqrt{3}}{2},\frac{-227}{16})$ and $(\frac{\sqrt{3}}{2},\frac{-227}{16})$

iv. $(\frac{2}{3},\frac{-34}{27})$

v. No inflection points because 0 is not in the domain of $h(x)$.

vi. No inflection points because 0 is not in the domain of $f(x)$.

vii. No inflection points.

viii. $(0,0)$

(e) Use the second derivative test to find the relative extrema of the following functions.

i. Minimum: $(1, \frac{-1}{2})$, Maximum: $(\frac{-2}{3}, \frac{49}{27})$
ii. Minima: $(0, -15)$, $(-1, -15)$, Maximum: $(\frac{-1}{2}, \frac{-239}{16})$
iii. $h(x)$ has a critical point when $x = 0$, and $h''(0) = 0$, so the second derivative test is inconclusive. By testing for increasing and decreasing intervals, we can show that $(0, 1)$ is a minimum.
iv. Minimum: $(\frac{1}{3}, \frac{-308}{27})$, Maximum: $(3, 44)$

6. Section 12.3 Graphing Functions

(a) Determine the vertical and horizontal asymptotes of the following functions.

i. No Asymptotes.
ii. No Asymptotes.
iii. Vertical Asymptote of $x = 0$ and Horizontal Asymptote of $y = 1$.
iv. Vertical Asymptote of $x = 0$, No Horizontal Asymptote.
v. No Asymptotes.
vi. No Asymptotes.

(b) Graph the following functions using the information you determine above.

i. $f(x) = 8x^2 - 12x + 10$
ii. $g(x) = \frac{1}{3}x^3 - 3x^2 - 16x + 3$
iii. $f(x) = x^4 - \frac{9}{2}x^2 - 8$
iv. $g(x) = x^3 - 2x^2 + 2x - 2$
v. $h(x) = 1 - \frac{1}{x}$
vi. $f(x) = \frac{x^2+1}{x}$
vii. $g(x) = x\sqrt{x}$
viii. $h(x) = x^{2/3}(x^2 - 1)$

7. Section 12.4 and 12.5 Optimization I and II

(a) Find the absolute maximum and minimum of the following functions on the given intervals.

i. Absolute maximum of 19 at $x = 2$ and an Absolute minimum of -1 at $x = 0$ or $x = 3$.
ii. Absolute maximum of 16 at $x = -2$ and an Absolute minimum of -1 at $x = -1$.
iii. Absolute maximum of 3 at $x = 2$ and an Absolute minimum of $\frac{5}{3}$ at $x = 4$.
iv. Absolute maximum of 0 at $x = 0$ and an Absolute minimum of -1.96 at $x = 3$.

(b) page 791 problems 41,45,47 (Answers in the back of your book)

(c) page 792 problem 53 (Answer in the back of your book)

(d) page 803-804 problems 3,5,9,21 (Answers in the back of your book)

i. For problem 3, minimize the function $C(x) = 6x + \frac{2400}{x}$, where x is the length of the fences made of pine board fencing and $C(x)$ is the cost of the project.

ii. For problem 5, maximize the function $V(x) = 4x^3 - 32x + 16$, where x is the dimensions of the corners cut from the tin sheet and $V(x)$ is the volume of the box created by folding the sides up. See Example 2 on page 798 for a picture.

iii. For problem 9, maximize the function $V(x) = 108x^2 - 4x^3$, where x is the length of the side of the package as shown in the figure, and $V(x)$ is the volume of the box.

iv. For problem 21, minimize the function $C(x) = 2.5\sqrt{9000000 - x^2} - 1.5x + 1500$, where x is as given in the figure and $C(x)$ is the cost of laying the cable.