

MATH 229 Review Sheet for the Final Exam

COMMENT: Do **NOT** try to do every problem. It would not be a good studying strategy. Try at least 1 from each section. Do more if you are struggling. When you feel you have command over a certain set of problems, feel free to move on. This review sheet is a good representative of what problems will be like on the final. This does not mean that the problems on the final will be exactly like these, but the concepts will be the same.

1. Find an equation of the line that passes through the given points.

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|----------------------------|------------------------------|
| a. $(4, -3)$ and $(-2, 7)$ | b. $(-6, -3)$ and $(2, -1)$ |
| c. $(3, 2)$ and $(-7, 10)$ | d. $(6.5, -5)$ and $(7, -3)$ |
| e. $(3, 6)$ and $(3, -4)$ | f. $(-2, -7)$ and $(4, -7)$ |

2. page 58 #15, #21, #37, #39

3. page 51 #13, #21, #23

4. page 591 #3, #5, #7, #17, #19, #23, #29, #37, #49, #53, #55, #57

5. page 607-608 #1, #5, #29, #33, #35, #43, #49, #55, #57

6. page 629 #9, #15, #25, #27

7. Use the definition of the derivative $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of the following functions.

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|----------------------|--------------------------|
| a. $f(x) = 3$ | b. $f(x) = x^2 - 3x + 1$ |
| c. $f(x) = \sqrt{x}$ | d. $f(x) = -3x^2 + 4$ |

8. Find the first derivative of the following functions.

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|----------------------------------|---|---------------------------------------|
| a. $f(x) = -2x^3$ | b. $f(x) = (x^3 - 13x + 5)(x^2 + 1)$ | c. $f(x) = 15x^{3/5}$ |
| d. $f(x) = -3\sqrt[3]{x}$ | e. $f(t) = \frac{5}{t^3} - \frac{3}{t^2} + \frac{1}{t}$ | f. $f(x) = \ln(e^x)$ |
| g. $f(x) = (2x^2 + 3)\sqrt{x}$ | h. $f(t) = (1 + \sqrt{t})(t^3 + 4t)$ | i. $f(x) = \frac{1}{x^2+1}$ |
| j. $f(u) = \frac{u}{1-u^2}$ | k. $f(x) = \frac{x^2+2}{x^2+3x-3}$ | l. $f(x) = \frac{x^3-2}{x^2+1}$ |
| m. $f(t) = 2(t^4 - t)^5$ | n. $f(t) = \frac{1}{t^3-1}$ | o. $f(x) = \frac{1}{\sqrt{3x^3-1}}$ |
| p. $f(t) = \frac{(2t-1)^2}{t+1}$ | q. $f(x) = \frac{2x^3}{(x^2-1)^3}$ | r. $f(t) = (t^{-1} - t^{-2})^3$ |
| s. $f(x) = x^2e^x$ | t. $g(x) = \frac{e^x}{x^2}$ | u. $h(u) = e^{-u^2}$ |
| v. $f(t) = t^2e^{-2t}$ | w. $f(x) = \frac{e^x+e^{-x}}{2}$ | x. $g(x) = (2 - e^{8x})^{12}$ |
| y. $g(x) = \frac{\ln x}{x^2}$ | z. $h(x) = \ln(x^3 - x)$ | α . $f(t) = \ln(x-3)^2(x+1)^7$ |

9. Find the slope and the equation of the tangent line to the previous functions at the following points.

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|----------------|-------------|-------------------|--------------|
| a. $(-1, 2)$ | b. $(0, 5)$ | c. $(32, 120)$ | d. $(1, -3)$ |
| e. $(1, 3)$ | f. $(2, 2)$ | g. $(1/4, 25/16)$ | h. $(1, 10)$ |
| i. $(3, 1/10)$ | j. $(0, 0)$ | | |

10. Find an equation of the tangent line to $f(x) = x^2e^{-\frac{x^2}{2}}$ at the point $(1, \frac{1}{\sqrt{e}})$.

11. Find an equation of the tangent line to $g(x) = \ln(2x^3 + 1)$ at the point $(1, \ln 3)$.
12. Find the points where the tangent line to the given function has the given slope m .
- a. $f(x) = x^2 - 3x + 4$; $m = 3$ b. $f(x) = x^3 - 2x^2 - 2$; $m = 0$
c. $f(x) = \frac{-2}{x}$; $m = 8$ d. $f(x) = \frac{1}{3}x^3 - 3x^2 + 8x - 10$; $m = 0$
13. Find $\frac{dy}{du}$, $\frac{du}{dx}$ and $\frac{dy}{dx}$.
- a. $y = u^3$ and $u = 2x^2 + 3x$ b. $y = u^{3/2}$ and $u = x^3 + 4$
c. $y = \sqrt{u} + \frac{1}{\sqrt{u}}$ and $u = x^2 + 2x$
14. Let $h(x) = f(g(x))$ and let $f'(2) = -4$, $f(3) = 2$, $g(5) = 2$ and $g'(5) = -1$, find $h'(5)$.
15. Let $k(x) = g(f(x))$ and let $f'(1) = 3$, $f(1) = -7$, $g(3) = 14$ and $g'(-7) = 1/9$, find $k'(1)$.
16. page 691 5-13 odd (You needn't do any of the graphing.)
17. Find the first and second derivatives of the following functions.
- a. $f(x) = \frac{2x}{x^2-1}$ b. $g(x) = \sqrt{x-3}$ c. $f(x) = (x^2 - 3)^2(x + 2)$
d. $f(x) = \frac{x-1}{x+1}$ e. $f(x) = 3x^{-3} - 4x^{-2} + x^{-1} - 2$
18. Find the intervals where the following functions are increasing and decreasing.
- a. $f(x) = 8x^2 - 12x + 10$ b. $g(x) = \frac{1}{3}x^3 - 3x^2 - 16x + 3$
c. $f(x) = x^4 - \frac{9}{2}x^2 - 8$ d. $f(x) = e^{-\frac{x^2}{2}}$
e. $g(x) = \frac{\ln x}{x^2}$
19. Find the critical points of the functions above and determine whether they are relative maximums, relative minimums, or neither.
20. Determine the intervals on which the functions above are concave up and concave down.
21. Determine the inflection points of the functions above.
22. Use the second derivative test to find the relative extrema of the following functions.
- a. $f(x) = x^3 - \frac{1}{2}x^2 - 2x + 1$ b. $g(x) = x^4 + 2x^3 + x^2 - 15$
c. $h(x) = \frac{1}{x^4+1}$ d. $f(x) = x^3 + 4x^2 - 3x - 10$
23. Graph the following functions using the information you determine above.

a. $f(x) = 8x^2 - 12x + 10$ b. $g(x) = \frac{1}{3}x^3 - 3x^2 - 16x + 3$
 c. $f(x) = x^4 - \frac{9}{2}x^2 - 8$ d. $f(x) = e^{-\frac{x^2}{2}}$
 e. $g(x) = \frac{\ln x}{x^2}$

24. Find the absolute maximum and minimum of the following functions on the given intervals.

a. $f(x) = x^3 + 3x^2 - 1; [-3, 2]$ b. $f(x) = 3x^4 + 4x^3; [-2, 1]$
 c. $f(x) = \frac{x+1}{x-1}; [2, 4]$ d. $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}; [0, 3]$

25. page 791 problems 41,45,47

26. page 794 problem 53

a. $\int x^3 dx$ b. $\int \frac{1}{\sqrt[3]{x^2}} dx$ c. $\int (3e^x - \frac{1}{x} + 3x^2) dx$
 d. $\int \frac{3}{x^4} dx$ e. $\int 8 dx$ f. $\int (x + \frac{1}{x}) dx$

27. Find $f(x)$ by solving the initial value problem.

a. $f'(x) = 3x^2 - x, f(1) = \frac{3}{2}$ b. $f'(x) = e^x - 4x^3, f(0) = 3$
 c. $f'(x) = 2x + \frac{1}{x}, f(1) = -3$

28. Page 901 problem 3.

29. Find the area of the region under the graph of the function f on the interval $[a, b]$.

a. $f(x) = x^2; [0, 4]$ b. $f(x) = e^x - x; [1, 2]$
 c. $f(x) = x + 3; [0, 2]$ d. $f(x) = \frac{1}{\sqrt{x}}; [1, 9]$

30. Evaluate the following definite integrals.

a. $\int_1^4 (2x - 3) dx$ b. $\int_{-1}^3 x^2 dx$ c. $\int_1^4 (x + \frac{1}{x}) dx$
 d. $\int_1^8 2x^{1/3} dx$ e. $\int_{-2}^2 (x^2 + 1) dx$ f. $\int_1^2 (1 + \frac{1}{x} + \frac{1}{x^2}) dx$

31. Find the average value of the following functions over the given interval.

a. $f(x) = 3x - 1; [1, 3]$ b. $f(x) = e^x + x; [0, 3]$ c. $f(x) = x^4 - 2x^2 + x; [-1, 1]$

32. Find the area of the region bounded below by the graph of each function and above by the x -axis from $x = a$ to $x = b$.

a. $f(x) = x^2 - 4; a = -2, b = 2$ b. $f(x) = -\sqrt{x}; a = 0, b = 4$

33. Sketch the graphs of the functions f and g and find the area of the region enclosed by these graphs between the vertical lines $x = a$ to $x = b$.
- $f(x) = 2x - 4$; $g(x) = x^2 - 4$; $a = 0$, $b = 2$
 - $f(x) = -x^2 + 2x + 3$; $g(x) = -x + 3$; $a = 0$, $b = 2$
 - $f(x) = x^2 + 1$; $g(x) = \frac{1}{3}x^3$; $a = -1$, $b = 2$
 - $f(x) = 4 - x^2$; $g(x) = x^2 - 4$; $a = -2$, $b = 2$
34. Sketch the graph and find the area of the region completely enclosed by the graphs of f and g .
- $f(x) = x^3$ and $g(x) = x^2$
 - $f(x) = x^3$ and $g(x) = x$
 - $f(x) = 4 - x^2$; $g(x) = x + 2$
35. page 949-950 #3, #5, #7
36. page 950 #11, #12, #16, #17
37. page 79 #1-#5 odd
38. page 92-93 #27, #37, #39, #43
39. page 106 #15, #19, #23
40. page 117 #19
41. page 132 #7, #17, #19