How to simplify a difference quotient, and some mistakes that I saw in the homework.

I noticed some mistakes in the homework that I want to discuss, and since I won't see you before the exam, I'm sending you an email to give you the chance to see your mistakes.

I'm going to do an example 4 times. The first three times I'll make mistakes. The last time I'll do it right. That way you can see what was going wrong. Mistakes are highlighted in red. Given the function $f(x) = 2x^2 - x + 1$, find

$$\frac{f(a+h) - f(a)}{h}.$$

1. (Attempt 1)

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{2(a+h)^2 - (a+h) + 1 - 2a^2 \color{red}{-a+1}}{h} \\ &= \frac{2(a^2+2ah+h^2) - a - h + 1 - 2a^2 \color{red}{-a+1}}{h}\end{aligned}$$

   Notice what happened. I didn't distribute the negative to all of $f(a)$ and so I didn't end up with the right sign on $-a+1$. Putting parentheses around $f(a)$ will prevent this. Notice the parenthases in green below.

2. (Attempt 2)

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{2(a+h)^2 - (a+h) + 1 - \color{green}{(}2a^2 - a + 1\color{green}{)}}{h} \\ &= \frac{2(a^2+2ah+h^2) - a - h + 1 - 2a^2 + a - 1}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - a - h + 1 - 2a^2 + a - 1}{h} \\ &= \frac{4ah + 2h^2 - \color{red}{\not h}}{\color{red}{\not h}} \\ &= \color{red}{4ah + 2h^2}\end{aligned}$$

   In this attempt the mistake that was made, was thinking that I could cancel the h on the bottom with the h on the top. This doesn't work. We must cancel an h in all terms.

3. (Attempt 3)

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{2(a+h)^2-(a+h)+1-(2a^2-a+1)}{h} \\
&= \frac{2(a^2+2ah+h^2)-a-h+1-2a^2+a-1}{h} \\
&= \frac{2a^2+4ah+2h^2-a-h+1-2a^2+a-1}{h} \\
&= \frac{4ah+2h^2-h}{h} \\
&= 4a+2h
\end{aligned}$$

The mistake in this attempt is that I cancel an h in each term, but the last term somehow disappears. Let's do this last step correctly.

4. (Attempt 4)

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{2(a+h)^2-(a+h)+1-(2a^2-a+1)}{h} \\
&= \frac{2(a^2+2ah+h^2)-a-h+1-2a^2+a-1}{h} \\
&= \frac{2a^2+4ah+2h^2-a-h+1-2a^2+a-1}{h} \\
&= \frac{4ah+2h^2-h}{h} \\
&= \frac{4ah}{h}+\frac{2h^2}{h}-\frac{h}{h} \\
&= 4a+2h-1
\end{aligned}$$

I hope you see the common mistakes now and that you understand how to do this type of a problem. Email me with questions if you have them.