MATH 261
Exam 3
Review Sheet

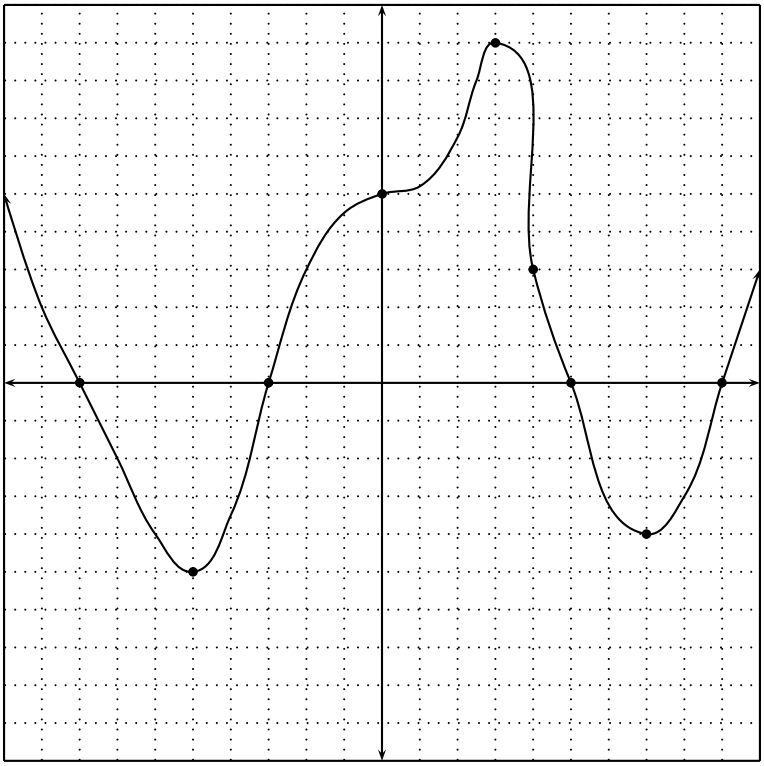
This review sheet is intended to remind you of the concepts that you are expected to understand for the exam. It is by no means a complete representation of what could be on the exam. You are responsible for everything discussed in the notes, on labs and in the suggested homework exercises.

1. Use the graph below to answer the following questions.

   (a) Suppose the graph below is a graph of f. State the intervals where f is increasing, decreasing, concave up and concave down. State the locations of the local maxima and local minima and inflection points.

   (b) Suppose the graph below is a graph of f'. State the intervals where f is increasing, decreasing, concave up and concave down. State the locations of the local maxima and local minima and inflection points.

   (c) Suppose the graph below is a graph of f''. State the intervals where f is concave up and concave down. State the locations of any inflection points.



2. Suppose the previous graph is a graph of $g(x)$. Answer the following questions.

   (a) Where is $g'(x) = 0$?

(b) Where is $g''(x) = 0$?

3. Let $f(x) = x^4 - 27x + 22$. Show that $f(x)$ satisfies the hypotheses of Rolle's theorem on $[0, 3]$ and find a number c that satisfies the conclusion of Rolle's Theorem.

4. Let $g(x) = \frac{x-3}{2x+1}$. Show that $g(x)$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$ and find a number c that satisfies the conclusion of the MVT.

5. Suppose a Vespa is driving along a straight highway in North Dakota for 60 seconds. Every 10 seconds the position, velocity and acceleration are measured. Distance is measured in feet and time is measured in seconds. The data are in the following chart. Use the chart to answer the following questions. Be sure to state the theorem that you are using to support your answer.

time	0	10	20	30	40	50	60
$s(t)$	0	20	50	90	130	190	210
$v(t)$	0	5	5	5.5	4	3	0
$a(t)$	1	5	7	10	2	2	1

   (a) Is there a time when the Vespa is 75 feet from the start?

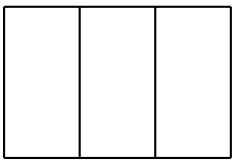
   (b) Is there a time when the Vespa is traveling 6 feet per second?

   (c) Is there a time when the acceleration of the Vespa is 0?

6. Find the absolute extrema of $f(x) = \frac{x^2}{x^2+4}$ on $[-1, 3]$.

7. Find the absolute extrema of $g(x) = x^3 - 4x^2 + 5x - 12$ on $[0, 3]$.

8. A farmer has 4000 feet of fence and wants to fence off a rectangular plot of land for her livestock that looks like the picture below. What should the dimensions of the rectangle be if she wants to maximize area?

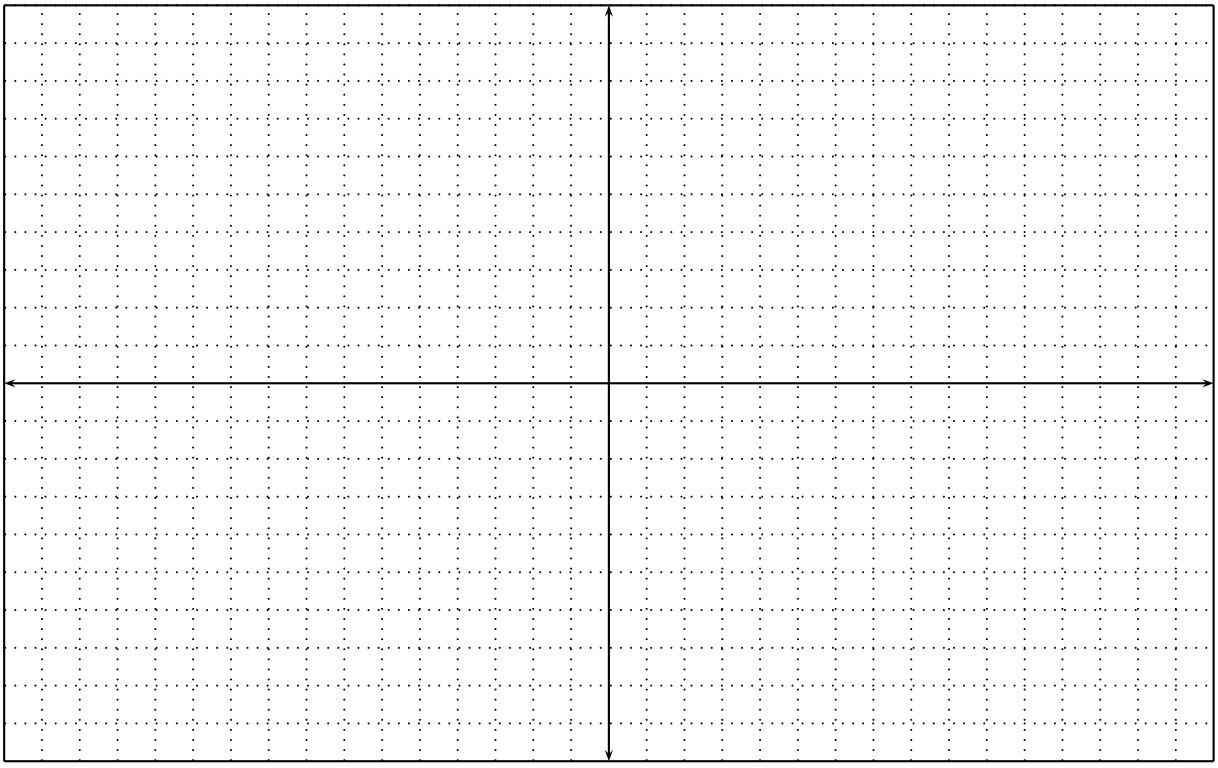


9. A cylindrical container without a top is to be constructed from tin. The volume of the container needs to be 8 cubic inches. If there is no waste in construction, find the dimensions of the container that requires the least amount of material. (Hint: minimize surface area)

10. A company that conducts bus tours found that when the price was \$9 per person, the average number of customers was 1000 per week. When the company reduced the price to \$7 per person, the average number of customers increased to 1500 per week. Assuming that the demand function is linear, what price should be charged to obtain the maximum weekly revenue?

11. A bulk grocer charges \$0.35 per pound for flour if you buy 10 pounds of flour. The grocer will reduce the price by \$0.02 per pound each time you buy 5 more pounds of flour. At what price is the grocer's revenue on flour the highest?

12. Use the grid below to graph a function $f(x)$ that has the following properties.
Domain: $(-\infty, -5) \cup (-5, \infty)$
x-intercepts: $(-13, 0)$, $(-8, 0)$, $(-2, 0)$ $(12, 0)$
y-intercept: $(0, 1)$ Increasing on $(-11, -5)$, $(-5, 2)$, and $(5, 10)$
Decreasing on $(-\infty, -11)$, $(2, 5)$, and $(10, \infty)$
Concave up on $(-12, -5)$, $(-2, 0)$, and $(12, \infty)$
Concave down on $(-\infty, -12)$, $(-5, -2)$, $(0, 5)$, and $(5, 12)$
Local Max: $(2, 6)$ and $(10, 4)$
Local Min: $(-11, -5)$ and $(5, 1)$
Inflection points $(-12, -2)$, $(-2, 0)$, $(0, 1)$ and $(12, 0)$
$\lim_{x \to -\infty} f(x) = 2$, $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -5^-} f(x) = \infty$, $\lim_{x \to -5^+} f(x) = -\infty$



13. For each of the following functions, determine the domain of the function, the x and y intercepts, where the function is increasing and decreasing, any local extrema, where the function is concave up and concave down, find any inflection points and find any asymptotes. Use this information to sketch a graph of the function.

    (a) $f(x) = x^4 - 12x^2$

    (b) $g(x) = \frac{x+2}{x-3}$

14. The position function of a particle traveling in a straight line is given by $s(t) = -2t^3 - 3t^2 + t - 14$. Find the velocity of the particle at time t. Find the acceleration of the particle at time t. Find the position, velocity and acceleration after 2 seconds. Find the time on the interval $[0, 4]$ when speed is greatest. (Remember speed is $|v(t)|$.)

15. Use Newton's Method to approximate $\sqrt{7}$ to 4 decimal places.

16. Use Newton's Method to approximate the real root of $x^3 + 2x - 7$ to 5 decimal places. Use $x_1 = 2$.