

Check the last two examples from p. 65 in Wed. Guided Notes:

Ex. 6 Find the Banzhaf power distribution of the weighted voting system [9: 5, 5, 4, 2, 1].

$$P_1: \frac{8}{24}; P_2: \frac{8}{24}; P_3: \frac{8}{24}; P_4: 0; P_5: 0.$$

The winning coalitions (with critical players underlined) are: $A: \frac{1}{3}; B: \frac{1}{3}; C: \frac{1}{3}; D: 0$. $\{\underline{P}_1, \underline{P}_3\}$, $\{\underline{P}_2, \underline{P}_3\}$, $\{\underline{P}_1, \underline{P}_2, P_5\}$, $\{P_1, P_2, P_3\}$, $\{\underline{P}_1, \underline{P}_2, P_4\}$, $\{\underline{P}_1, \underline{P}_2, P_5\}$, $\{\underline{P}_1, \underline{P}_3, P_4\}$, $\{\underline{P}_1, \underline{P}_3, P_5\}$, $\{\underline{P}_2, \underline{P}_3, P_4\}$, $\{\underline{P}_2, \underline{P}_3, P_5\}$, $\{P_1, P_2, P_3, P_4\}$, $\{P_1, P_2, P_3, P_5\}$, $\{\underline{P}_1, \underline{P}_2, P_4, P_5\}$, $\{\underline{P}_1, \underline{P}_3, P_4, P_5\}$, $\{\underline{P}_2, \underline{P}_3, P_4, P_5\}$, $\{P_1, P_2, P_3, P_4, P_5\}$.

Ex. 7 (Review) Write the explicit formula for the following sequence: 900, 90, 9, 0.9 ...

What's missing in this formula? $S_n = 900(0.1)$

Solutions to Wed. assigned Banzhaf Power Index problems

12.

(e) $P_1: \frac{1}{2}; P_2: \frac{3}{10}; P_3: \frac{1}{10}; P_4: \frac{1}{10}.$

The winning coalitions (with critical players underlined) are: $\{\underline{P}_1, \underline{P}_2\}$, $\{\underline{P}_1, \underline{P}_2, P_3\}$, $\{\underline{P}_1, \underline{P}_2, P_4\}$, $\{\underline{P}_1, \underline{P}_3, P_4\}$, $\{\underline{P}_1, P_2, P_3, P_4\}$.

15. (a) $P_1: \frac{8}{24}; P_2: \frac{6}{24}; P_3: \frac{4}{24}; P_4: \frac{4}{24}; P_5: \frac{2}{24}.$

The winning coalitions (with critical players underlined) are: $\{\underline{P}_1, \underline{P}_2, P_3\}$, $\{P_1, P_2, P_4\}$, $\{\underline{P}_1, \underline{P}_2, P_5\}$, $\{\underline{P}_1, \underline{P}_3, P_4\}$, $\{\underline{P}_1, P_2, P_3, P_4\}$, $\{\underline{P}_1, \underline{P}_2, P_3, P_5\}$, $\{\underline{P}_1, \underline{P}_2, P_4, P_5\}$, $\{\underline{P}_1, \underline{P}_3, P_4, P_5\}$, $\{\underline{P}_2, \underline{P}_3, P_4, P_5\}$, $\{P_1, P_2, P_3, P_4, P_5\}$.

(b) $P_1: \frac{7}{19}; P_2: \frac{5}{19}; P_3: \frac{3}{19}; P_4: \frac{3}{19}; P_5: \frac{1}{19}.$

The quota is one more than in (a), so some winning coalitions may now be losing coalitions. For the ones that are still winning, any players that were critical in (a) will still be critical, and there may be additional critical players. A quick check shows that $\{P_1, P_2, P_5\}$, $\{P_1, P_3, P_4\}$, and $\{P_2, P_3, P_4, P_5\}$ are now losing coalitions (they all have exactly 10 votes). The winning coalitions (with critical players underlined) are: $\{\underline{P}_1, \underline{P}_2, \underline{P}_3\}$, $\{\underline{P}_1, \underline{P}_2, \underline{P}_4\}$, $\{\underline{P}_1, \underline{P}_2, P_3, P_4\}$, $\{\underline{P}_1, \underline{P}_2, P_3, P_5\}$, $\{\underline{P}_1, \underline{P}_2, P_4, P_5\}$, $\{\underline{P}_1, \underline{P}_3, P_4, P_5\}$, $\{\underline{P}_2, \underline{P}_3, P_4, P_5\}$.

Sequential Coalition: Coalition taking into account the order the players joined the coalition.

The symbol $\langle P_1, P_2, P_3 \rangle$ is a sequential coalition of three players.

$N!$: the number of sequential coalitions possible with N players.

Pivotal Player: The player who, when he joins the sequential coalition turns the coalition from a losing coalition to a winning coalition.

- Those who came before are not powerful enough
- Those who came after are not absolutely necessary for the coalition to win.

There is only one pivotal player per coalition.

Shapley-Shubic Power Index

Step 1: make a list of all sequential coalitions containing all N players
(there should be $N!$ of them)

Step 2: in each sequential coalition determine the pivotal player.

Step 3: count the total number of times P is pivotal. Call this value S.

Shapley-Shubic Power Index of P = $\frac{S}{N!}$

The value of the Shapley-Shubic Power Index can be the same as the value of the Banzhaf Power Index for the same weighted voting system, BUT USUALLY IT IS NOT.

Ex. 1 Consider the weighted voting system [8: 7, 6, 2]

- Write down all the sequential coalitions involving all three players.
- In each sequential coalition above, circle the pivotal player.
- Find the Shapley-Shubik power distribution for this weighted voting system.

Ex 2 Find the Shapley-Shubik power distribution of the weighted voting system [60: 32, 30, 28]

Ex. 3 What type of voting system is most likely used to rank the Dragons second in the NSIC wrestling coaches poll with 42 points and one first place vote on 10/24?

Assignment:

No Class on Monday the 27th Professor Harms is gone
Project 2 is Due by Tues. Oct. 28th to MacLean 375H

For Wed Oct. 29th:

Read Chapter 2 pp.61-71
Do #23, 28, 45, 48 & 54 on pp. 74-76