Chapter 5 Notes from________

Start-up problem
#38. a & b on p. 190

Length of a path: the number of edges in the path.

X, Y, Z, Z is a path of length 3.

Use the following graph for Example 1

1.a. Find a path of length 4 from B to D that is.

b. How many paths are there from A to D?

c. Name a bridge in the graph.

Summary of Euler Theorems

<table>
<thead>
<tr>
<th>Number of Odd Vertices</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Graph has one or more Euler Circuits</td>
</tr>
<tr>
<td>2</td>
<td>Graph has one or more Euler Paths but no Euler Circuits</td>
</tr>
<tr>
<td>4, 6, 8, ...</td>
<td>Graph has neither Euler Paths nor Euler Circuits</td>
</tr>
<tr>
<td>1, 3, 5, ...</td>
<td>Impossible. No such graphs exist</td>
</tr>
</tbody>
</table>
A true story: Seven gold coins were offered as a prize for the first person who could find a way to walk across each one of the seven bridges of Konigsberg without recrossing any and return to the original starting point. A smaller prize (5 gold coins) was offered for anyone who can cross each of the seven bridges exactly once without necessarily returning to the starting point. No one has ever collected either prize.

Figure 5-14 of your textbook, (reproduced below) shows a map of the Konigsberg bridge area. Notice that the river splits and there are two islands in the middle of the split. The seven bridges are shown in part (a) below by the dark black lines.

EX. 2 Fill in parts (b) and (c) of Figure 5-12 to show the situation simplified as a “graph” representation of the land masses (represented by vertices) and the bridges (represented by edges).

Figure 5-12

Notice that all 4 of the vertices in part (c) above are vertices with odd degree. When the mathematician Euler proved that a graph with more than 2 odd vertices would not have an Euler circuit, he proved that it was not possible to solve the Konigsberg Bridge challenge. No one will ever be able to win the prize.

This is an example of how real path or schedule problems can be solved (or proved impossible to solve) using the mathematics of graph theory.
EX. 3 Another Example of Representing a Map as a Graph
A security patrol going down the middle of the street effectively patrols both sides of the street at once. The mail carrier, however, must deliver to both sides of the street whenever there are houses on both sides of the street.

Steps for Fleury’s Algorithm used to find an Euler circuit or Euler path:

Step 1: First make sure that the graph is connected and all the vertices have even degree or has 2 vertices of odd degree.

Step 2: If there is a pair of odd degree vertices pick any one of those as your starting point, or if all vertices are of even degree any pick any starting point you would like.

Step 3. When you have a choice, always choose to travel along an edge that is not a bridge of the yet-to-be-traveled part of the graph. (Don’t burn your bridges!!!!)

Step 4. Label the edges in the order in which you travel them.

Step 5. When you can’t travel any more, stop. You are done!

**Example 4: Apply the Fleury Algorithm to each of the figures below.**

For problem (a) number your edges  For problem (b) write out the Euler path that you find.
Since graphs with more than 2 odd vertices will have no Euler Circuit or Path, finding an optimal route in these cases requires that we re-cross some of the edges. The question becomes how can we cover all the edges of the graph with only re-crossing the minimum number of edges. This process is called “Eulerizing” a Graph. When Eulerizing a graph, we add edges that meet two requirements:

(a) only add edges that duplicate already existing edges
(b) only add edges beginning and ending at odd vertices.

5. The following is not a legal Eulerization of the original graph. What is wrong with it?

6. The following is not an efficient Eulerization of the original graph. What makes it inefficient?

7. Make a legal Eulerization of this graph:
A **Semi-Eulerization** of a graph makes all but two of the vertices even, so that the graph can be modified to make a Euler Path (as opposed to Euler Circuit).

The difference is that the path does not have to start and end at the same vertex.

The same rules apply to creating Semi-Eulerizations as to creating Eulerizations.

Remember that the Euler Path for such a graph must **start at one of the odd vertices and end at the other odd vertex**.

An **optimal eulerization** is adding the minimum number of edges to the graph following the two requirements (only add edges that duplicate already existing edges & only add edges beginning and ending at odd vertices) so that all the vertices are of even degree.

**Ex. 8** Find an optimal eulerization for each of the graphs below.

a) ![Graph A](image)

b) ![Graph B](image)
Example 9

Identify odd vertices

Add minimum number of edges

Check your work above with Figure 5-22 on p. 180

Assignment due for 11/14

Read the first part of Chapter 5 pp. 175-185
Do #1, 4, 9, 11, 17, 41, 42 on pp. 185-190

Rework problems on separate sheet of paper for any problem you had lost points on in Test 3 by this Friday for a 10 pt assignment.