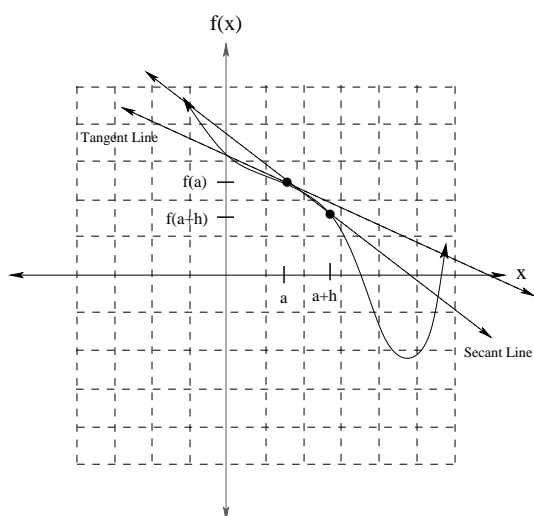


Math 261  
Secant Lines and Tangent Lines Handout



- A *secant line* meets the graph of a function at two points. The slope of the secant line through the points  $P(a, f(a))$  and  $Q(a+h, f(a+h))$  is equal to the *average rate of change* of the function  $f$  over the interval  $[a, a+h]$ . For example, the slope of a secant line to a position (displacement) graph is the *average speed* of the object being described over the time period between the two points on the graph intersecting the line.

- A *tangent line* meets the graph of a function at a point  $P(a, f(a))$ , and has slope equal to the *instantaneous rate of change* of the function  $f$  at the point  $P$ . For example, the slope of a tangent line to a position (displacement) graph is the *instantaneous velocity* of the object being described at that particular point in time.

- The slope of the tangent line to a function  $f(x)$  at the point  $P(a, f(a))$  (that is, when  $x = a$ ) is given by:

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}, \text{ when this limit exists.}$$

**Example:** If  $f(x) = -2x^2 + 8x$ , and  $x = a$ , then:

$$m_a = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{-2(a+h)^2 + 8(a+h) - (-2a^2 + 8a)}{h} = \lim_{h \rightarrow 0} \frac{-2(a^2 + 2ah + h^2) + 8a + 8h + 2a^2 - 8a}{h} =$$

$$\lim_{h \rightarrow 0} \frac{-2a^2 - 4ah - 2h^2 + 8a + 8h + 2a^2 - 8a}{h} = \lim_{h \rightarrow 0} \frac{-4ah - 2h^2 + 8h}{h} = \lim_{h \rightarrow 0} -4a - 2h + 8 = -4a + 8$$

In particular, if  $a = 1$ , then  $m_a = -4 + 8 = 4$

- To find an equation for a tangent line to a function  $f$  at a point  $P(a, f(a))$ , we use the point  $P$  together with the slope  $m_a$ .

**Example:** given the function  $f(x) = -2x^2 + 8x$ , and  $x = 1$ ,  $(1, f(1)) = (1, 6)$ , and  $m = 4$ , so the equation of the tangent line at this point is given by:  $y - 6 = 4(x - 1)$ , or  $y = 4x + 2$ .

