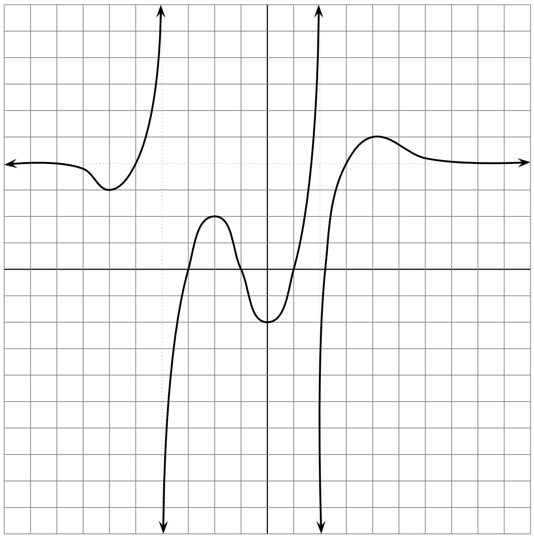
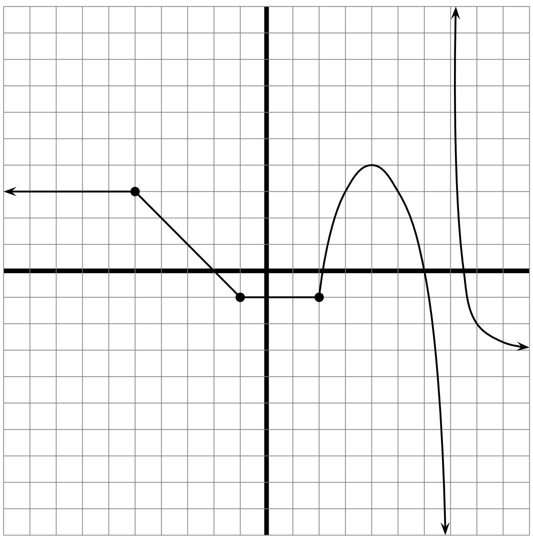
Math 261 Exam 3 Practice Problems

1. The graph of f is given. Answer the following questions.



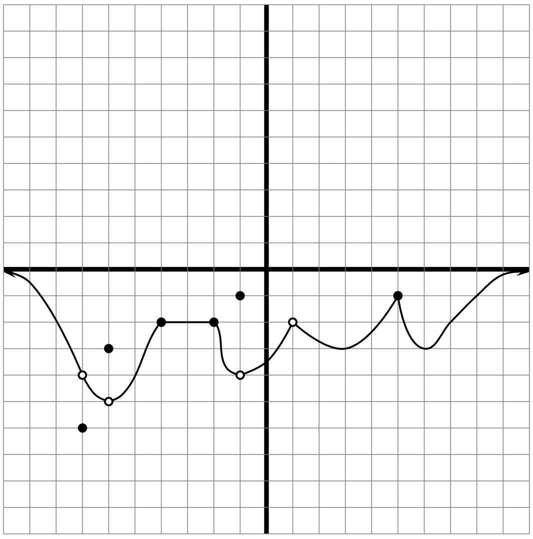
   (a) Find the intervals where f is increasing.
   (b) Find the intervals where f is decreasing.
   (c) Find any local extrema.
   (d) Find the intervals where f is concave up.
   (e) Find the intervals where f is concave down.
   (f) Find any inflection points.
   (g) Find any asymptotes.
   (h) Find the x- and y-intercepts.
   (i) Sketch a possible graph for $f'(x)$.

2. The graph of f is given below. Answer the following questions.



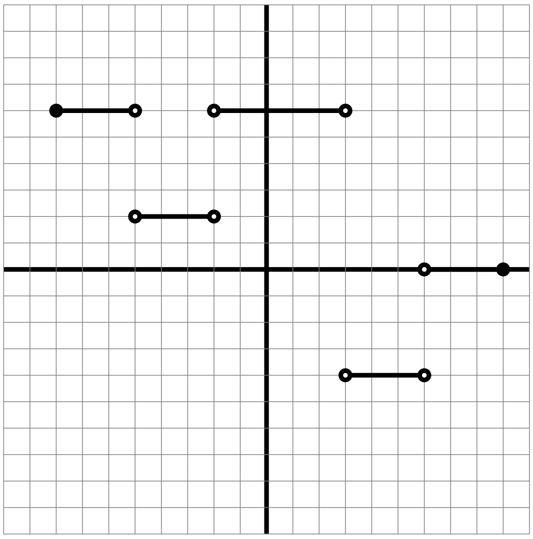
   (a) Find the intervals where f is increasing.
   (b) Find the intervals where f is decreasing.
   (c) Find the intervals where f is constant.
   (d) Find where there are any local extrema.
   (e) Find the intervals where f is concave up.
   (f) Find the intervals where f is concave down.
   (g) Find any inflection points.
   (h) Find any asymptotes.
   (i) Find the x- and y-intercepts.
   (j) Sketch a possible graph for $f'(x)$.

3. The graph of f is given below. Answer the following questions.



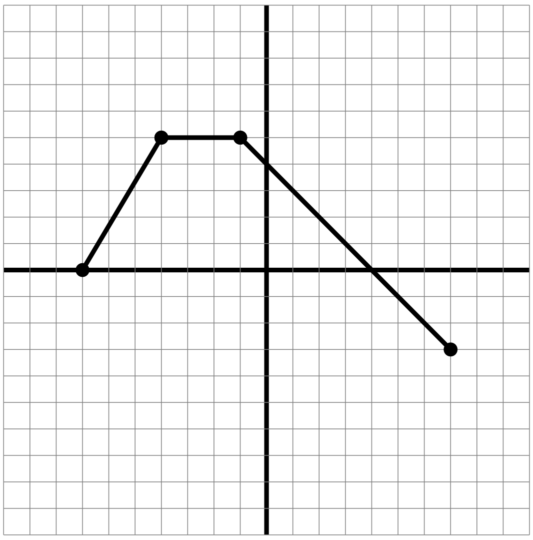
   (a) Find the intervals where f is increasing.
   (b) Find the intervals where f is decreasing.
   (c) Find the intervals where f is constant.
   (d) Find all local maximums.
   (e) Find the location of all local maximums.
   (f) Find all local minimums.
   (g) Find the location of all local minimums.
   (h) Find the absolute maximum and its location, if it exists.
   (i) Find the absolute minimum and its location, if it exists.
   (j) Find the absolute maximum on the inteval $[3, 8]$, if it exists.
   (k) Find the absolute minimum on the interval $[3, 8]$, if it exists.

4. The graph of f' (the derivative of f) is shown. Answer the following questions.



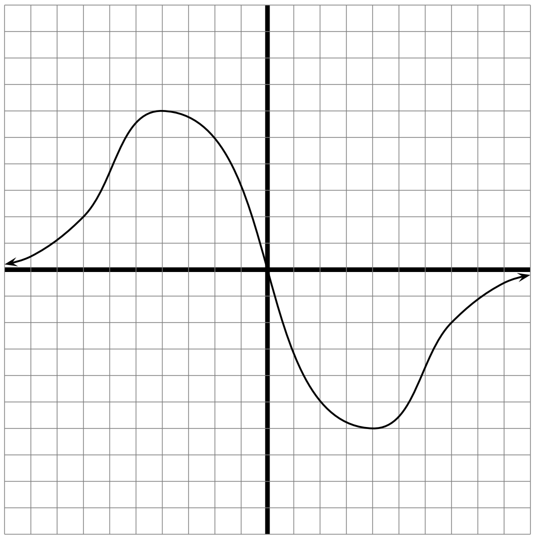
   (a) Find where f is increasing.
   (b) Find where f is decreasing.
   (c) Find where f is constant.
   (d) Find where f has a local maximum.
   (e) Find where f has a local minimum.
   (f) Sketch a possible graph of f.

5. The graph of f' is shown. Answer the following questions.



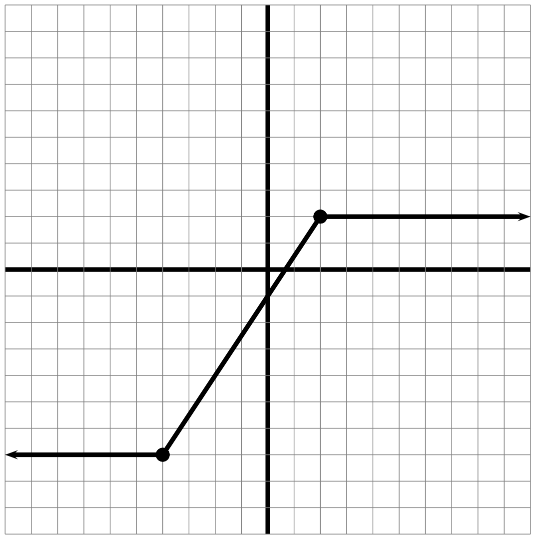
   (a) Find where f is increasing.
   (b) Find where f is decreasing.
   (c) Find where f has a local maximum.
   (d) Find where f has a local minimum.
   (e) Find where f is concave up.
   (f) Find where f is concave down.
   (g) Sketch a possible graph of f.
   (h) Sketch a graph of f''.

6. The graph of f' is shown. Answer the following questions.



   (a) Find where f is increasing.
   (b) Find where f is decreasing.
   (c) Find where f has a local maximum.
   (d) Find where f has a local minimum.
   (e) Find where f' is increasing.
   (f) Find where f' is decreasing.
   (g) Find where f' has a local maximum.
   (h) Find where f' has a local minimum.
   (i) Find where f is concave up.
   (j) Find where f is concave down.
   (k) Sketch a possible graph of f.
   (l) Find where f'' is positive.
   (m) Find where f'' is negative.
   (n) Sketch a possible graph of f''.

7. The graph of f'' is shown. Answer the following questions.



(a) Sketch a possible graph of f'.

(b) Where is f concave up?

(c) Where is f concave down?

(d) Where is f' linear?

8. A particle is moving along a straight line during a sixty-second time interval. Every ten seconds the position, velocity, and acceleration of the particle are measured, and recorded in the table below. For each of the following questions, answer the question and justify your answer.

t (seconds)	0	10	20	30	40	50	60
$x(t)$ (feet)	0	10	12	15	30	35	60
$v(t)$ (ft/s)	0	0.1	0	-2	0	0	0
$a(t)$ (ft/s^2)	20	10	5	1	0	5	10

(a) Is there a time when $x(t) = 5$ feet?

(b) Is there a time when $x(t) = 70$ feet?

(c) Is there a time when $v(t) = 1$ ft/sec?

(d) Is there a time when $v(t) = 0.2$ ft/sec?

(e) Is there a time when $v(t) = -1$ ft/sec?

(f) What is the largest velocity that you can justify from the data in the table?

(g) Is there a time when $a(t) = 0$ ft/sec^2?

(h) Is there a time when $a(t) = 15$ ft/sec^2?

(i) Is there a time when $a(t) = -0.2$ ft/sec^2?

(j) Is there a time when $a(t) = -10$ ft/sec^2?

9. Find the absolute maximum and the absolute minimum of each of the following functions on the given domain.

(a) $f(x) = 3x^2 + 2x - 5$ on the interval $[-5, -1]$

(b) $f(x) = 2x^2 - 7x + 1$ on the interval $[0, 9]$

(c) $f(x) = x^3 - 9x + 5$ on the interval $[-2, 5]$

(d) $f(x) = \dfrac{x+3}{2x-1}$ on the interval $[1, 10]$

(e) $f(x) = \dfrac{x^2+2}{1-x}$ on the interval $[-5, 0]$

10. A field that is to be used for a play area for a large dog is to be fenced in. The field is adjacent to a house, and the side with the house will need no fence. If 500 feet of fence is available, find the dimensions of the field that maximum the area in which the dog can play in.

11. Find a number c in the given interval that satisfies the Mean Value Theorem for the function and interval given, or explain why the Mean Value Theorem does not apply.

(a) $f(x) = x^3 - x$ on $[-1, 1]$

(b) $f(x) = 2x^3 + x^2 - x - 1$ on $[0, 2]$

(c) $f(x) = \dfrac{x+1}{x-1}$ on $[2, 5]$

(d) $f(x) = \dfrac{x+1}{x-1}$ on $[-1, 2]$

(e) $f(x) = 1 + \sqrt[3]{x-1}$ on $[2, 9]$

(f) $g(x) = |x-1|$ on $[0, 3]$

(g) $g(x) = |x-1|$ on $[-5, 0]$

(h) $h(\theta) = \sin\theta + \cos\theta$ on $[0, 2\pi]$

(i) $h(t) = \dfrac{2}{t^2}$ on $[1, 4]$

12. Show that of all rectangles with perimeter k (a fixed constant), the one with the maximum area is a square.

13. Sketch a graph which satisfies the following descriptions.

    domain is $\{x|x \neq -5 \text{ and } x \neq 5\}$
    y-intercept is $(0, 2)$
    x-intercepts are $(-3, 0)$, $(3, 0)$, and $(7, 0)$
    horizontal asymptote is $y = 3$
    vertical asymptotes are $x = 5$ and $x = -5$
    increasing on $(-\infty, -5)$
    increasing on $(-5, 0)$
    decreasing on $(0, 5)$
    increasing on $(5, \infty)$
    max at $(0, 2)$
    no mins
    concave up on $(-\infty, -5)$
    concave down on $(-5, 5)$
    concave down on $(5, \infty)$
    no points of inflection

14. Sketch a graph which satisfies the following descriptions.

    Domain is all real numbers, continuous for all real numbers.
    $\lim_{x\to-\infty} f(x) = 0$
    $\lim_{x\to\infty} f(x) = 3$
    $f'(x) > 0$ for $x < -5$ and $1 < x < 7$
    $f'(x) < 0$ for $-5 < x < 1$ and $x > 7$
    $f'(1) = f'(7) = 0$
    $f'(-5)$ does not exist
    $f(-5) = 5$, $f(1) = -2$, and $f(7) = 4$
    $f''(x) > 0$ for $x < -5$, $-5 < x < 4$, and $x > 8$
    $f''(x) < 0$ for $4 < x < 8$
    $f''(4) = f''(8) = 0$
    $f''(-5)$ does not exist
    $f(4) = 1$ and $f(8) = 3.5$

15. Sketch the following functions using information on the domain, the intercepts, the intervals where the function is increasing or decreasing, any local maximum or minimum, the intervals where it is concave up or concave down, any inflection points, and any asymptotes.

    (a) $P(x) = 3x^2 - 6x - 5$
    (b) $f(x) = \dfrac{x+2}{x-1}$
    (c) $P(x) = x^3 - 3x^2 + 5x - 4$
    (d) $f(x) = \dfrac{1}{12}x^4 - \dfrac{1}{6}x^3 - 3x^2 - 1$
    (e) $f(x) = \dfrac{x+1}{(x-2)(x+3)}$
    (f) $f(x) = 3x^2 - 9$
    (g) $f(x) = 2x^3 - 15x^2 + 36$
    (h) $f(x) = x^4 - 4x + 2$
    (i) $f(x) = (x^2 - 1)^{\frac{4}{3}}$
    (j) $f(x) = \dfrac{x+3}{x-2}$
    (k) $f(x) = \dfrac{2x^2 - 1}{x-2}$
    (l) $f(x) = x + \dfrac{1}{x}$
    (m) $f(x) = \sin(x) + \cos(x)$ on $[-2\pi, 2\pi]$

16. Find any local extrema of the following functions. Classify each as a maximum or a minimum.

    (a) $f(x) = x^3 - 7x^2 - 5x + 6$
    (b) $f(x) = x^3\sqrt{x} - 14x^2 + 10$
    (c) $f(x) = 3x^5 - 65x^3 + 540x - 100$

17. Solve the following Optimization problems:

    (a) The owner of a ranch has 3000 yards of material to enclose a pasture along a straight stretch of river. If fencing is not required along the river, find the dimensions of the pasture of largest area that he can enclose.
    (b) Postal regulations require a parcel sent through US mail to have a combined length and girth of at most 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail.
    (c) The owner of a yacht charges \$600 per person if exactly 20 people sign up for a cruise. If more than 20 sign up for the cruise (up to a maximum capacity of 90), then the fare for each passenger is disounted by \$4 for each additional passenger beyond 20. Assuming that at least 20 people sign up, determine the number or passengers that will maximize the revenue for the cruise and find the fare for each passenger.