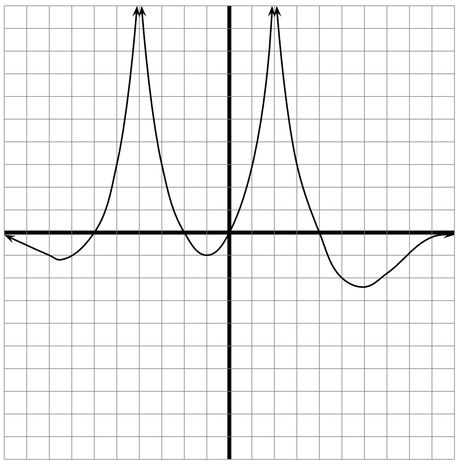
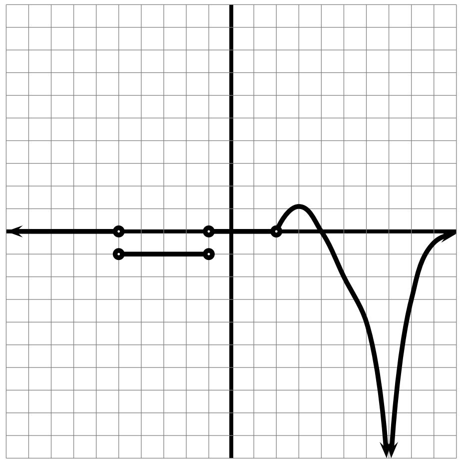
Math 261 Exam 3 Practice Problems - Solutions

1. (a) $(-6,-4)$, $(-4,-2)$, $(0,2)$, $(2,4)$
   (b) $(-\infty,-6)$, $(-2,0)$, $(4,\infty)$
   (c) local min 3 (at $x=-6$)
   local max 2 (at $x=-2$)
   local min -2 (at $x=0$)
   local max 5 (at $x=4$)
   (d) $(-6.7,-4)$, $(-1.3,-1)$, $(-0.7,2)$, $(5.3,\infty)$
   (e) $(-\infty,-6.7)$, $(-4,-1.3)$, $(-1,-0.7)$, $(2,5.3)$
   (f) $(-6.7,3.6)$, $(-1.3,1)$, $(-1,0)$, $(-0.7,-1)$, $(5.3,-4.5)$
   (g) vertical asymptotes $x=-4$ and $x=2$
   horizontal asymptote $y=4$
   (h) x-intercepts $(-3,0),(-1,0),(1,0),(2.2,0)$
   y-intercepts $(0,-2)$
   (i)

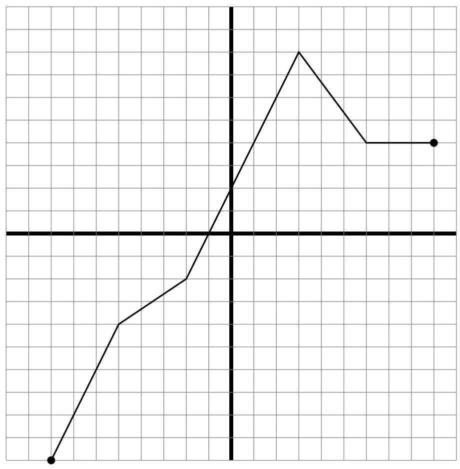


2. (a) $(-\infty,-5)\,,(-1,2),(2,4)$
   (b) $(-\infty,-5)\,,(-5,-1),(-1,2),(4,7),(7,\infty)$
   (c) $(-\infty,-5)\,,(-1,2)$
   (d) local mins for $x\in(-\infty,-5)$ and $x\in[-1,2]$
   local maxs for $x=4$ and $x\in(-\infty,-5]$ and $x\in(-1,2)$
   (e) $(7,\infty)$
   (f) $(2,7)$
   (g) none (only changes concavity at an asymptote)
   (h) horizontal asymptotes $y=3$ and $y=-3$
   vertical asymptote $x=7$
   (i) x-intercepts $(-2,0),(2.2,0),(7.5,0)$
   y-intercept $(0,-1)$
   (j)



3. (a) $(-6,-4),(-4,-2)$, $(-1,1),(3,5),(6,\infty)$
   (b) $(-\infty,-7),(-7,-6)$, $(-4,-2),(-2,-1)$, $(1,3),(5,6)$
   (c) $(-4,-2)$
   (d) $-3,-2,-1$
   (e) $x=-6$, $x\in[-4,-2]$, $x=-1, x=5$
   (f) $-6,-2,-3$
   (g) $x=-7$, $x\in(-4,-2)$, $x=3, x=6$
   (h) absolute max does not exist
   (i) absolute min is -6 at $x=-7$
   (j) absolute max is -1
   (k) absolute min is -3

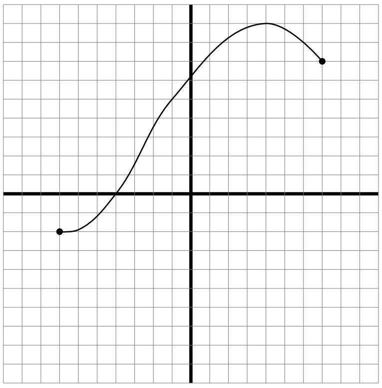
4. (a) $(-8,-5),(-5,-2),(-2,3)$
   (b) $(3,6)$
   (c) $(6,9)$
   (d) at $x=3$
   (e) for $x\in(6,9)$
   (f) The y-scale is by 3's.



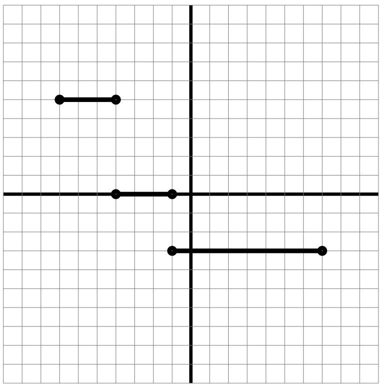
Note: The whole graph can be shifted vertically, but must maintain this piecewise linear shape with these slopes.

5. (a) $(-7, 4)$
   (b) $(4, 7)$
   (c) at $x = 4$
   (d) nowhere
   (e) $(-7, -4)$
   (f) $(-1, 7)$

   (g)



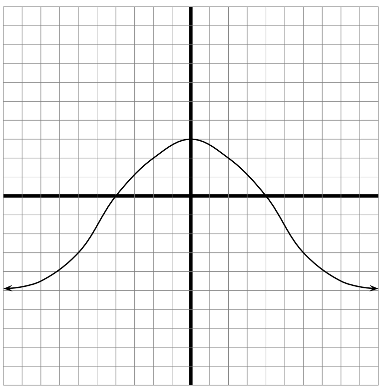
   (h)



The y-scale is three tick marks is one unit.

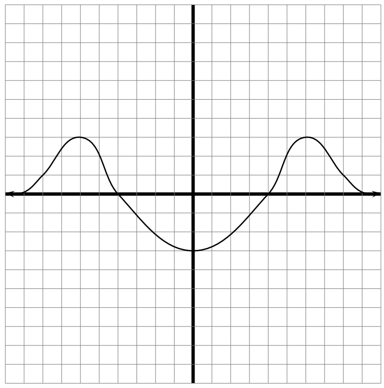
6. (a) $(-\infty, 0)$
   (b) $(0, \infty)$
   (c) at $x = 0$
   (d) nowhere
   (e) $(-\infty, -4), (4, \infty)$
   (f) $(-4, 4)$
   (g) at $x = -4$
   (h) at $x = 4$
   (i) $(-\infty, -4), (4, \infty)$
   (j) $(-4, 4)$

   (k)

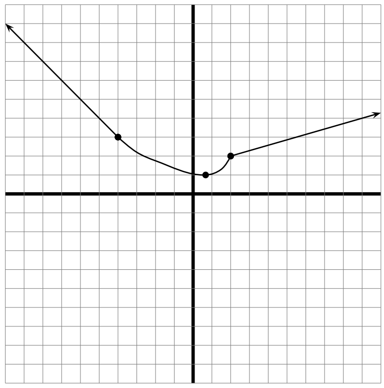


   (l) $(-\infty, -4), (4, \infty)$
   (m) $(-4, 4)$

   (n)



7. (a) The y-scale is by 7's.



   (b) $\left(\frac{2}{3}, \infty\right)$ (note: the $\frac{2}{3}$ is exact)
   (c) $\left(-\infty, \frac{2}{3}\right)$
   (d) f' is linear where its slope is constant, so where f'' is constant. So on the intervals $(-\infty, -4)$ and $(2, \infty)$.

8. (a) Yes. Apply the Intermediate Value Theorem (IVT) for $0 \le t \le 10$.
   (b) Unknown. Although the distances shown do not pass 70 feet, we don't know the particle passed that point and returned in the same 10-second interval.
   (c) Yes. Apply the Mean Value Theorem (MVT) for $0 \le t \le 60$.
   (d) Yes. Apply the MVT for $10 \le t \le 20$.
   (e) Yes. Apply the IVT for $20 \le t \le 30$ or $30 \le t \le 40$ or ...
   (f) $v(t) = 2.5$ feet/second by applying the MVT for $50 \le t \le 60$

(g) Yes. Apply the MVT for $0 \le t \le 40$ or $40 \le t \le 50$ or ...

(h) Yes. Apply the IVT for $0 \le t \le 10$

(i) Yes. Apply the MVT for $30 \le t \le 40$

(j) Unknown. Although it appears as though the acceleration will likely not be that far negative, we can't show that with isolated, discrete values. It's possible that the acceleration is varying wildly and quickly between the times measured.

9. (a) max 60, min -4

(b) max 100, min $-\dfrac{41}{8}$

(c) max 85, min $-6\sqrt{3}+5$

(d) max 4, min $\dfrac{13}{19}$

(e) max $\dfrac{9}{2}$, min $2\sqrt{3}-2$

10. The dimensions are 125×250 feet, where the longer side is parallel to the house.

11. (a) $c = \pm\dfrac{1}{\sqrt{3}}$

(b) $c = \dfrac{1+\sqrt{61}}{6}$

(c) $c = 3$

(d) The function is not continuous on $[-1, 2]$, so the MVT does not apply.

(e) $c = 1 + \dfrac{7\sqrt{7}}{3\sqrt{3}}$

(f) The function is not differentiable on (0,3), so the MVT does not apply.

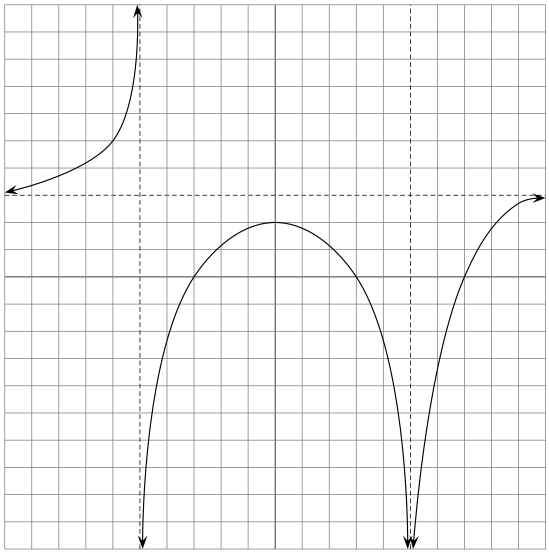
(g) c can be any value in $(-5, 0)$

(h) $\theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{5\pi}{4}$

(i) $c = \sqrt[3]{\dfrac{32}{5}}$

12. (Use the perimeter to get length in terms of width and k. Then give area as a function of width only. Take the derivative, find the critical numbers. The domain is $\left[0, \frac{k}{2}\right]$, so evaluate the area at the endpoints and at the critical numbers. You'll find the maximum area occurs when width=length=$\frac{1}{2}k$.)

13.



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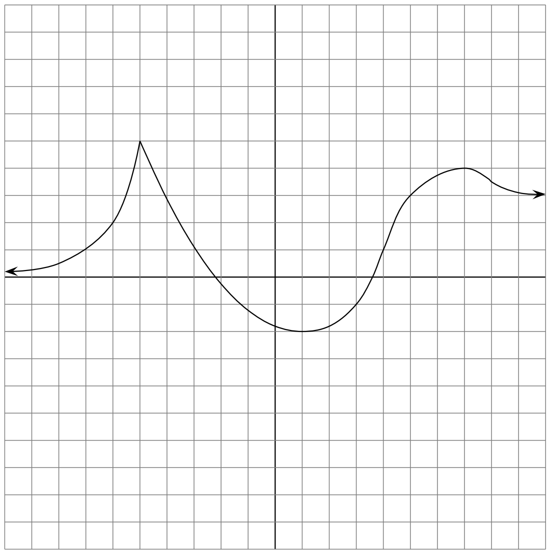


15. (a) Domain $(-\infty, \infty)$

y-intercept $(0, -5)$

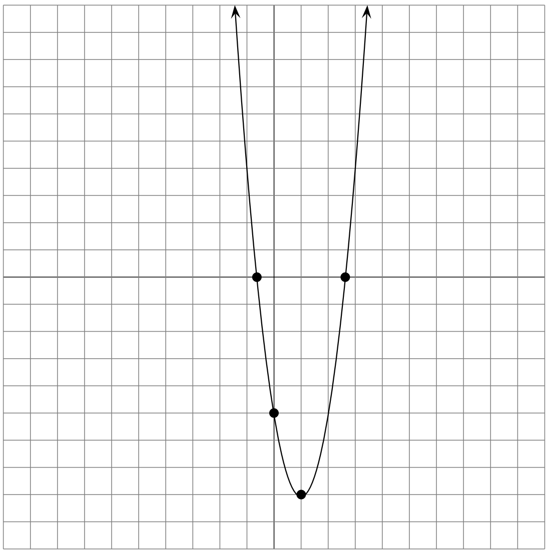
x-intercept $\left(\frac{3+2\sqrt{6}}{3}, 0\right)$, $\left(\frac{3-2\sqrt{6}}{3}, 0\right)$

Decreasing $(-\infty, 1)$

Increasing $(1, \infty)$

Minimum at $(1, -8)$

Concave up $(-\infty, \infty)$



(b) Domain $\{x|x \neq 1\}$

y-intercept $(0, -2)$

x-intercept $(-2, 0)$

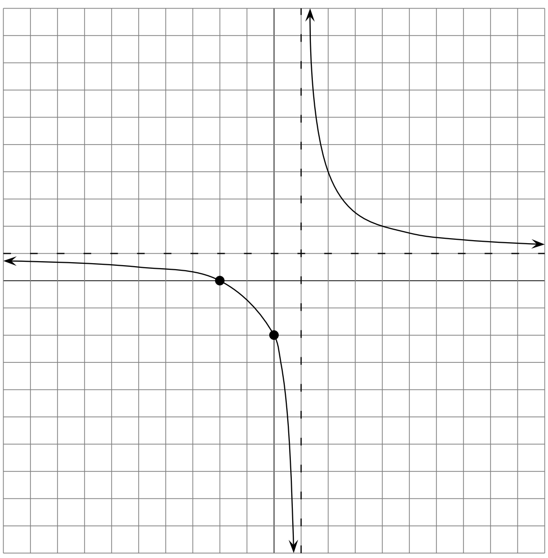
vertical asymptote $x = 1$

horizontal asymptote $y = 1$

Decreasing $(-\infty, 1), (1, \infty)$

Concave up $(-\infty, 1)$

Concave down $(1, \infty)$



(c) Domain $(-\infty, \infty)$

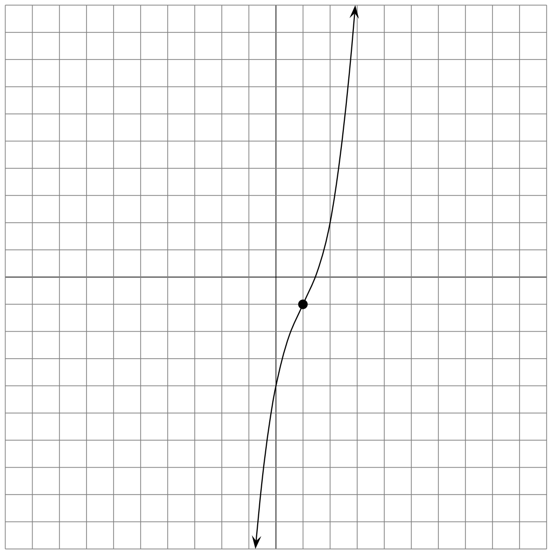
y-intercept $(0, -4)$

Increasing $(-\infty, \infty)$

Concave down $(-\infty, 1)$

Concave up $(1, \infty)$

Point of inflection $(1, -1)$



(d) Domain $(-\infty, \infty)$

y-intercept $(0, -1)$

Decreasing $\left(-\infty, \frac{3-3\sqrt{17}}{4}\right)$

Increasing $\left(\frac{3-3\sqrt{17}}{4}, 0\right)$

Decreasing $\left(0, \frac{3+3\sqrt{17}}{4}\right)$

Increasing $\left(\frac{3+3\sqrt{17}}{4}, \infty\right)$

Minimum at approximately $(-2.3, -12.8)$

Maximum at $(0, -1)$

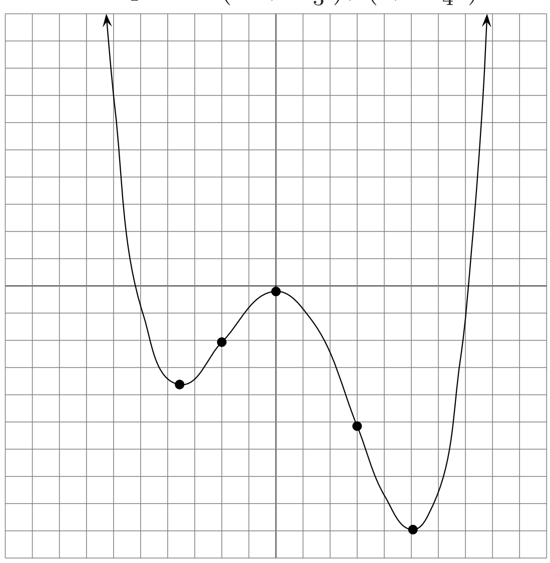
Minimum at approximately $(3.8, -36.6)$

Concave up $(-\infty, -2)$

Concave down $(-2, 3)$

Concave up $(3, \infty)$

Inflection points $\left(-2, -\frac{31}{3}\right)$, $\left(3, -\frac{103}{4}\right)$



The y-scale is one tick mark is 5 units.

(e) domain $\{x | x \neq 2 \text{ and } x \neq -3\}$

x-intercept $(-1, 0)$, y-intercept $\left(0, -\dfrac{1}{6}\right)$

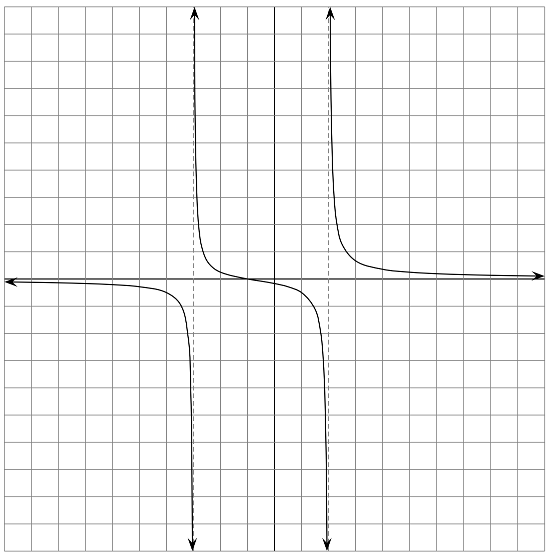
horizontal asymptote $y = 0$

vertical asymptotes $x = -3$ and $x = 2$

decreasing $(-\infty, -3)$, $(-3, 2)$, and $(2, \infty)$

no extrema

$\longrightarrow$ The second derivative is of the form of a third-degree polynomial divided by $(x^2 + x - 6)^3$. Because we can not solve cubic equations in general (and for this case), not all of the critical numbers of the second derivative can be solved for. Skip the concavity and inflection points for this problem.



(f) domain is all real numbers

x=intercepts $\pm\sqrt{3}$, y-intercept -9

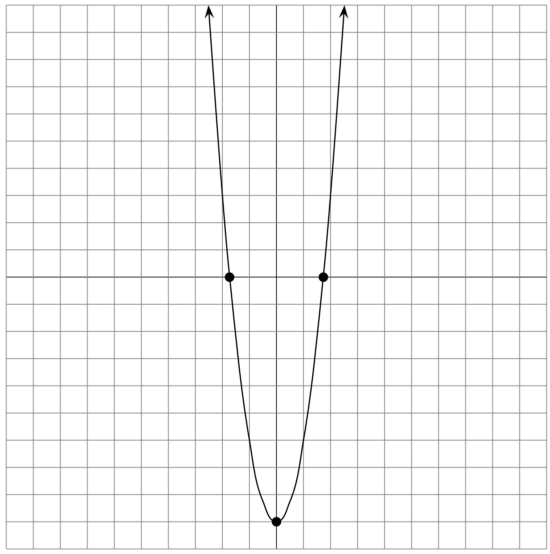
no asymptotes

decreasing $(-\infty, 0)$
increasing $(0, \infty)$

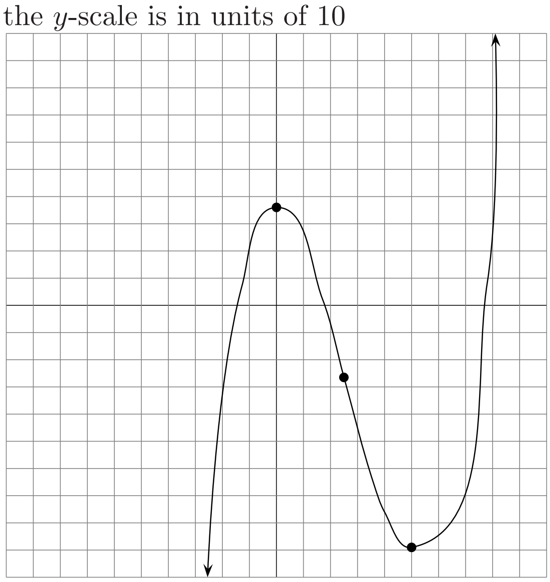
the point $(0, -9)$ is a local minimum

concave up on $(-\infty, \infty)$

no points of inflection



(g) domain is all real numbers
y-intercept 36
no asymptotes
increasing $(-\infty, 0)$
decreasing $(0, 5)$
increasing $(5, \infty)$
the point $(0, 36)$ is a local maximum
the point $(5, -89)$ is a local minimum
concave down on $(-\infty, \frac{5}{2})$,
concave up on $(\frac{5}{2}, \infty)$
point of inflection $\left(\frac{5}{2}, -\frac{53}{2}\right)$
the y-scale is in units of 10



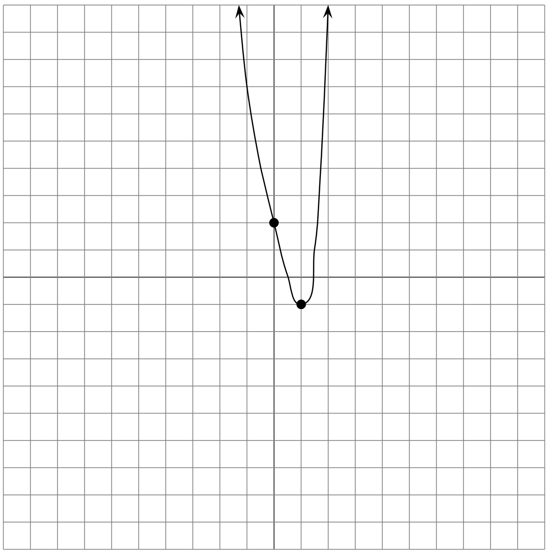
(h) domain is all real numbers
y-intercept 2
no asymptotes

decreasing $(-\infty, 1)$
increasing $(1, \infty)$

the point $(1, -1)$ is a local minimum

concave up on $(-\infty, \infty)$

no points of inflection



(i) domain is all real numbers

x=intercepts ± 1, y-intercept 1

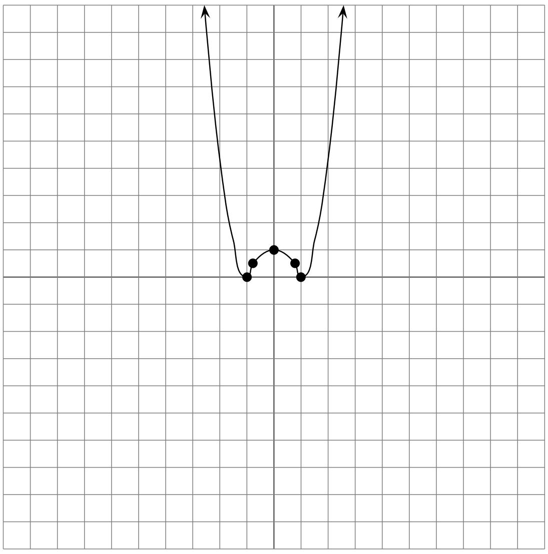
no asymptotes

decreasing $(-\infty, -1)$
increasing $(-1, 0)$
decreasing $(0, 1)$
increasing $(1, \infty)$

the point $(-1, 0)$ is a local minimum
the point $(0, 1)$ is a local maximum
the point $(1, 0)$ is a local minimum

concave up on $(-\infty, -1)$
concave up on $\left(-1, -\sqrt{\frac{5}{3}}\right)$
concave down on $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$
concave up on $\left(\sqrt{\frac{5}{3}}, 1\right)$
concave up on $(1, \infty)$

the points $\left(\pm\sqrt{\frac{3}{5}}, \frac{2}{5}\sqrt[3]{\frac{2}{5}}\right)$ are points of inflection



(j) domain $\{x | x \neq 2\}$

x=intercept -3, y-intercept $-\dfrac{3}{2}$

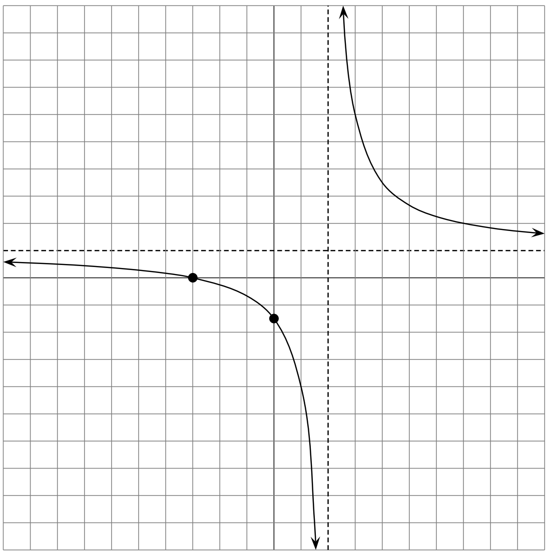
horizontal asymptote $y = 1$
vertical asymptote $x = 2$

decreasing $(-\infty, 2)$
decreasing $(2, \infty)$

no extremum

concave down on $(-\infty, 2)$
concave up on $(2, \infty)$

no points of inflection



(k) domain $\{x | x \neq 2\}$

x=intercepts $\pm\dfrac{1}{\sqrt{2}}$, y-intercept $\dfrac{1}{2}$

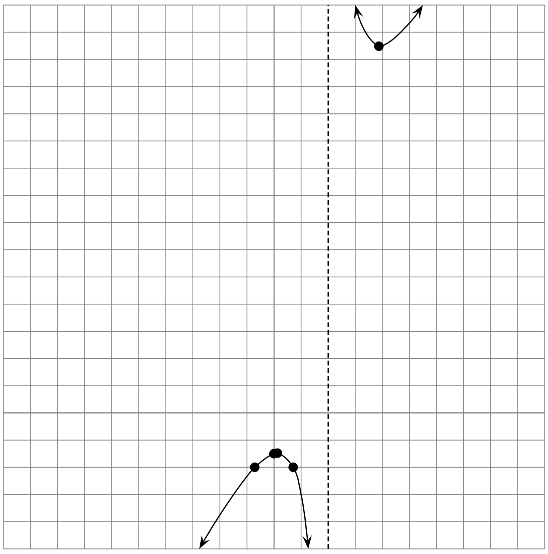
vertical asymptote $x = 2$
no horizontal asymptote

increasing $\left(-\infty, \frac{4-\sqrt{14}}{2}\right)$
decreasing $\left(\frac{4-\sqrt{14}}{2}, 2\right)$
decreasing $\left(2, \frac{4+\sqrt{14}}{2}\right)$
increasing $\left(\frac{4+\sqrt{14}}{2}, \infty\right)$

the point $\left(\frac{4-\sqrt{14}}{2}, 8-2\sqrt{14}\right)$ is a local maximum
the point $\left(\frac{4+\sqrt{14}}{2}, 8+2\sqrt{14}\right)$ is a local minimum

concave down on $(-\infty, 2)$
concave up on $(2, \infty)$

no points of inflection



(l) domain $\{x|x \neq 0\}$

no x=intercepts, no y-intercept

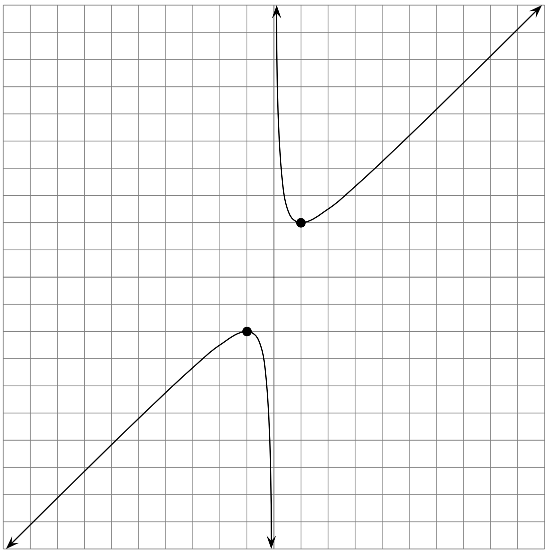
vertical asymptote $x = 0$
no horizontal asymptote

increasing $(-\infty, -1)$
decreasing $(-1, 0)$
decreasing $(0, 1)$
increasing $(1, \infty)$

the point $(-1, -2)$ is a local maximum
the point $(1, 2)$ is a local minimum

concave down on $(-\infty, 0)$
concave up on $(0, \infty)$

no points of inflection



(m) domain $[-2\pi, 2\pi]$

continuous on $[-2\pi, 2\pi]$

y-intercept $(0, 1)$

x-intercepts $\left(-\frac{5\pi}{4}, 0\right)$, $\left(-\frac{\pi}{4}, 0\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{7\pi}{4}, 0\right)$

increasing $\left[-2\pi, -\frac{7\pi}{4}\right)$

decreasing $\left(-\frac{7\pi}{4}, -\frac{-3\pi}{4}\right)$

increasing $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$

decreasing $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

increasing $\left(\frac{5\pi}{4}, 2\pi\right]$

local maximums $\left(-\frac{7\pi}{4}, \sqrt{2}\right)$ and $\left(\frac{\pi}{4}, \sqrt{2}\right)$

local minimums $\left(-\frac{3\pi}{4}, -\sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

concave down $\left[-2\pi, -\frac{5\pi}{4}\right)$

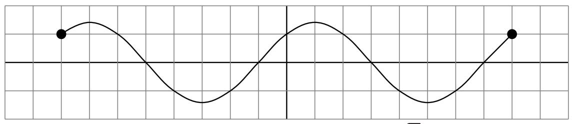
concave up $\left(-\frac{5\pi}{4}, -\frac{\pi}{4}\right)$

concave down $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$

concave up $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

concave down $\left(\frac{7\pi}{4}, 2\pi\right]$

inflection points $\left(-\frac{5\pi}{4}, 0\right)$, $\left(-\frac{\pi}{4}, 0\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{7\pi}{4}, 0\right)$



The x-scale is one tick mark is $\frac{\pi}{4}$ units

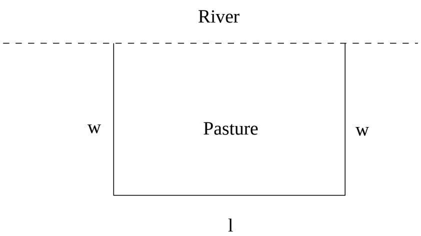
16. (a) Maximum is $\frac{185}{27}$ (at $x = -\frac{1}{3}$).
    Minimum is -69 (at $x = 5$).

    (b) Maximum is 10 (at $x = 0$).
    Minimum is -86 (at $x = 4$).

    (c) Maximum is -694 (at $x = -3$).
    Minimum is -756 (at $x = -2$).
    Maximum is 556 (at $x = 2$).
    Minimum is 494 (at $x = 3$).

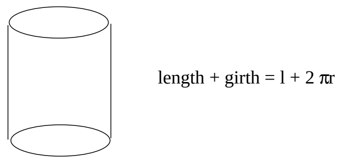
17. Solve the following Optimization problems:

    (a) The owner of a ranch has 3000 yards of material to enclose a pasture along a straight stretch of river. If fencing is not required along the river, find the dimensions of the pasture of largest area that he can enclose.



Let l be the width of the fence and w the width. Then $2w + l = 3000$, and $A = l \cdot w = w(3000 - 2w) = 3000w - 2w^2$. Therefore, $A'(w) = 3000 - 4w$, which as a critical when $3000 = 4w$, or when $w = 750$. Since $A''(w) = -4$, we see by the Second Derivative test that this is a local maximum. Finally, since $0 \le w \le 1500$, and the boundary values give pastures with zero area, the pasture of largest area has width 750 yards and length 1500 yards.

    (b) Postal regulations require a parcel sent through US mail to have a combined length and girth of at most 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail.



Notice that the girth of a cylindrical package is $2\pi r$. Therefore, $108 = l + 2\pi r$, or $l = 108 - 2\pi r$. The volume of a cylinder is $v = \pi r^2 l = \pi r^2(108 - 2\pi r) = 108\pi r^2 - 2\pi^2 r^3$.

Then $V'(r) = 216\pi r - 6\pi^2 r^2 = -6\pi r(\pi r - 36)$. Critical numbers: $r = 0$ and $r = \frac{36}{\pi}$.

Notice that $V''(r) = 216\pi - 12\pi^2 r$, which is positive when $r = 0$, and negaive when $r = \frac{36}{\pi}$. Therefore, the maximum volume parcel is one with a radius of $\frac{36}{\pi}$ inches and length $108 - 2\pi\frac{36}{\pi} = 108 - 72 = 36$ inches.

    (c) The owner of a yacht charges \$600 per person if exactly 20 people sign up for a cruise. If more than 20 sign up for the cruise (up to a maximum capacity of 90), then the fare for each passenger is disounted by \$4 for each additional passenger beyond 20. Assuming that at least 20 people sign up, find the number or passengers that will maximize the revenue for the cruise and the fare for each passenger.

Let x be the number of people *beyond* the original 20 people who sign up for the cruise. Then the total number of people who go is $20 + x$, where $0 \le x \le 70$, and the price per person for each passenger is $600 - 4x$. Hence the revenue for the cruise is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12000$.

Therefore, $R'(x) = -8x + 520$, which has a critical number when $-8x + 520 = 0$, or when $520 = 8x$, and $x = 65$. Moreover, $R''(x) = -8$, so this is a local minimum.

Finally, we can easily check to see that:

$R(0) = (20)(600) = \$12,000$

$R(65) = (85)(600 - 4 \cdot 65) = \$28,900$

$R(70) = (90)(600 - 4 \cdot 70) = \$27,200$

Hence the revenue for the cruise is maximized when $x = 65$. That is, when 85 people take the cruise. The fare for each passenger here is $600 - 4 \cdot 65) = \$340$.