13. Given that $h(x) = f(g(x))$ and the following table of values:

	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	7	-2	-1	2
4	3	1	0	2

Find the following:

(a) $(f+g)(1) = f(1) + g(1) = 7 + (-2) = 5$

(b) $(f+g)'(1) = f'(1) + g'(1) = -1 + 2 = 1$

(c) $(fg)(4) = f(4) \cdot g(4) = 3 \cdot 1 = 3$

(d) $(fg)'(4) = f'(4)g(4) + f(4)g'(4) = 0 + (3)(2) = 6$

(e) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = -\frac{7}{2}$

(f) $h'(4) = f'(g(4))g'(4) = f'(1)g'(4) = (-1)(2) = -2$

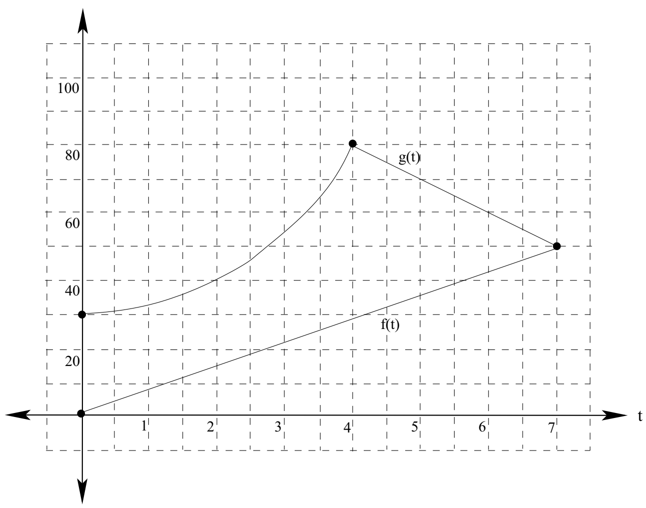
14. If $f(x) = \sqrt{3-2x}$, find the intervals where $f(x)$ is continuous, and find the intervals where $f(x)$ is differentiable.

First notice that f is continuous wherever it is defined, so we need $3 - 2x \geq 0$, or $x \leq \frac{3}{2}$, so f is continuous on $(-\infty, \frac{3}{2}]$.

Next, $f'(x) = \frac{1}{2}(3-2x)^{-\frac{1}{2}}(-2) = -(3-2x)^{\frac{1}{2}} = \frac{-1}{\sqrt{3-2x}}$

Therefore, f is differentiable when $3 - 2x > 0$, or when $x < \frac{3}{2}$. That is, on the interval $(-\infty, \frac{3}{2})$.

15. The total net value of two companies: Company A and Company B, in millions of dollars as a function of time (in years since the year 2000) is given by $f(t)$ and $g(t)$ respectively.



(a) How fast was company A's value growing in 2005? (include units for your answer)

Notice that the growth of company A in 2005 is given by the value of the derivative of f when $t = 5$, but since f is linear, its derivative is constant, so we have $f'(5) = \frac{50}{7} \approx 7.143$ in millions of dollars per year.

(b) What was the average rate of change for company B from 2000 through 2007?

This is given by the slope of the secant line between $(0, g(0))$ and $(7, g(7))$.

Here $m_{av} = \frac{50-30}{7-0} = \frac{20}{7} \approx 2.857$ in millions of dollars per year.

(c) Which company's value was growing faster in 2004?

This is a bit open to interpretation since there is a cusp point on $g(t)$ when $t = 4$. Looking backwards, the value of company B is increasing more steeply than that of company A, but the rate of change of company B switches from positive to negative at $t = 4$.

Since we usually view time as moving foreward, we will say that at the beginning of 2004, Company B's value is growing faster.

(d) Which company's value was growing faster in 2001?

The situation here is much clearer. Looking at the tangent lines to each of the functions when $t = 1$, we see that the tangent line to $f(t)$ has larger slope, so Company A's value is growing faster at the beginning of 2001.

(e) Which company would you rather own stock in, and why?

This question is a bit qualitative, since we cannit know what will happen in the future for thee companies, but it appears that Company A is steadily increasing in value, while Company B has topped out and is starting to decline. I would rather own stock in Company A.

16. Find the equation for the tangent line to $f(x) = \sqrt[3]{x-5}$ when $x = 13$. Then, use this tangent line to approximate $\sqrt[3]{10}$.

First notice that $f'(x) = \frac{1}{3}(x-5)^{-\frac{2}{3}}$, so the slope of the tangent line is $\frac{1}{3}(13-5)^{-\frac{2}{3}} = \frac{1}{3}(8)^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$.

Next, notice that $f(13) = \sqrt[3]{13-5} = \sqrt[3]{8} = 2$, so by point/slope, the tangent line is given by the equation $y - 2 = \frac{1}{12}(x-13)$, or $y = \frac{1}{12}x + \frac{11}{12}$.

Now, $f(15) = \sqrt[3]{15-5} = \sqrt[3]{10}$. Then, using our tangent line to approximate:

$\sqrt[3]{10} \approx \frac{1}{12}(15) + \frac{11}{12} = \frac{15}{12} + \frac{11}{12} = \frac{26}{12} \approx 2.1666667$.

(In fact, calculating directly, $\sqrt[3]{10} \approx 2.15443469$)

17. Find the equation of the tangent line to the graph of $f(x) = \sin(2x)$ when $x = \frac{\pi}{6}$

$f'(x) = 2\cos(2x)$, so $m = 2\cos\left(\frac{\pi}{3}\right) = 2 \cdot \frac{1}{2} = 1$

Also, $f(\frac{\pi}{6}) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the tangent line equation is given by $y - \frac{\sqrt{3}}{2} = x - \frac{\pi}{6}$, or $y = x + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

18. Find the tangent line to the graph of the relation $4xy - 4x^2 = y^2$ at the point $(1, 2)$

Differentiating implicitly, $4y + 4xy' - 8x = 2yy'$, so $4y - 8x = 2yy' - 4xy' = y'(2y - 4x)$.

Thus $y' = \frac{4y-8x}{2y-4x}$, which when $x = 1$ and $y = 2$ gives:$y' = \frac{4(2)-8}{2(2)-4} = \frac{0}{0}$ – uh oh!

The tangent line is undefined! There is no tangent line at this point.

19. Prove the Quotient Rule by applying the the Chain Rule to the general function $h(x) = f(x)[g(x)]^{-1}$.

If $h(x) = f(x)[g(x)]^{-1}$, then $h'(x) = f'(x)[g(x)]^{-1} + f(x) \cdot (-1)[g(x)]^{-2} \cdot g'(x) = \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2} = \frac{f'(x)g(x)}{g(x)^2} - \frac{f(x)g'(x)}{g(x)^2}$

20. A person flying a kite holds the string 5 feet above ground level, and the string is played out at a rate of 2 feet per second as the kite moves horizontally at an altitude of 105 feet. Assuming that there is no sag in the string, find the rate at which the kite is moving when a total of 125 feet of string has been let out.

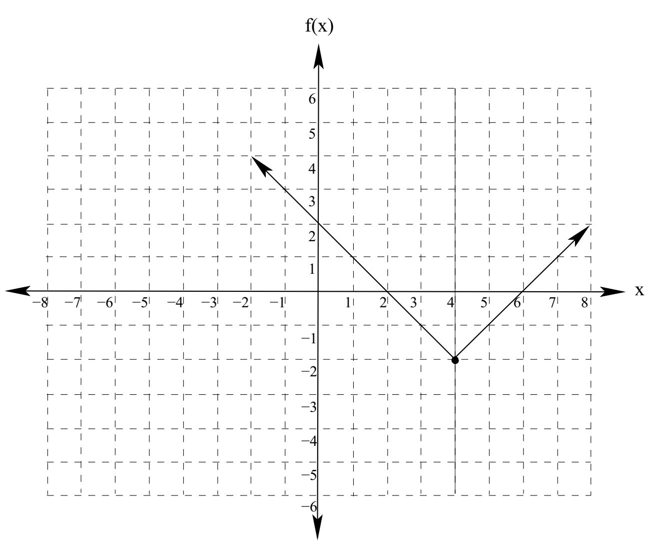
Let L be the length of the string. Let x be the horizontal distance of the kite from the place where the person is standing. Then, since the net height of the kite is 100 feet (105 - 5 feet), using the Pythagorean Theorem, we have $L^2 = x^2 + 100^2$.

Differentiating this implicitly, $2LL' = 2xx'$, or solving for x', $x' = \frac{2LL'}{2x} = \frac{LL'}{x}$.

Notice that $L = 125$, $L' = 2$, and $x = \sqrt{(125)^2 - 100^2} = 75$ at the time in question.

Therefore, $x' = \frac{(125)(2)}{75} = \frac{10}{3} \frac{ft}{sec}$.

21. Draw the graph of a function $f(x)$ that is continuous when $x = 4$, but is not differentiable when $x = 4$.



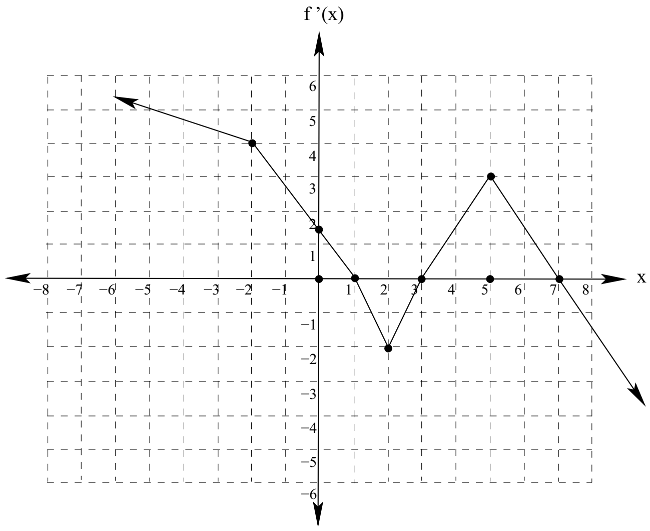
22. Find $f^{(6)}(x)$ if $f(x) = \cos(2x)$

    $f'(x) = -2\sin(2x)$; $f''(x) = -4\cos(2x)$; $f^{(3)}(x) = 8\sin(2x)$;
    $f^{(4)}(x) = 16\cos(2x)$; $f^{(5)}(x) = -32\sin(2x)$; $f^{(6)}(x) = -64\cos(2x) = -2^6\cos(2x)$

23. Find $f^{(21)}(x)$ if $f(x) = x^{20} + 7x^5 - 3x^3 - 1$

    $f^{(21)}(x) = 0$. Each time you differentiate, the power drops by 1, so the 20th derivative is a constant. Hence, the 21st derivative is zero.

24. The graph of f' is given. Answer the following questions.



    (a) Find the intervals where f is increasing.
    f is increasing whenever f' is positive. That is, on the intervals: $(-\infty, 1) \cup (3, 7)$.

    (b) Find the intervals where f is decreasing.
    f is increasing whenever f' is negative. That is, on the intervals: $(1, 3) \cup (7, \infty)$.

    (c) Find the location of all local maximums.
    There are local maxima whenever f' goes from positive to negative, so at $x = 1$ and $x = 7$

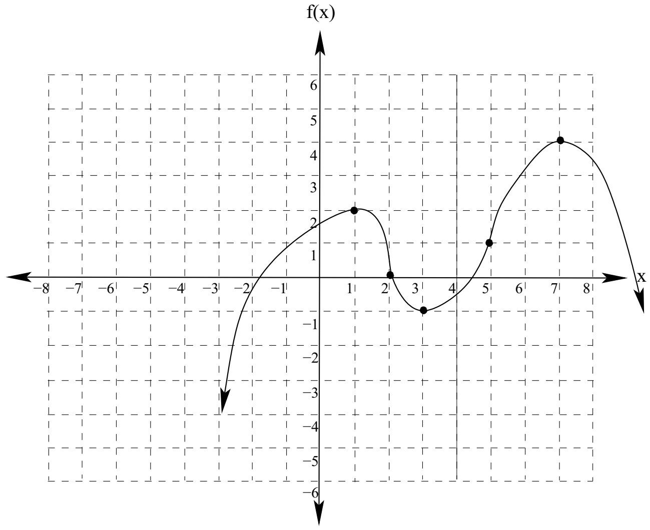
    (d) Find the location of all local minimums.
    There are local minima whenever f' goes from negative to positive, so at $x = 3$

    (e) Find the intervals where f is concave up.
    f is concave up whenever f' is increasing. That is, on the interval: $(2, 5)$

    (f) Find the intervals where f is concave down.
    f is concave down whenever f' is decreasing. That is, on the intervals: $(-\infty, 2) \cup (5, \infty)$.

    (g) Find any inflection points.
    Inflection points occur when the f changes concavity, so at: $x = 2$ and at $x = 5$

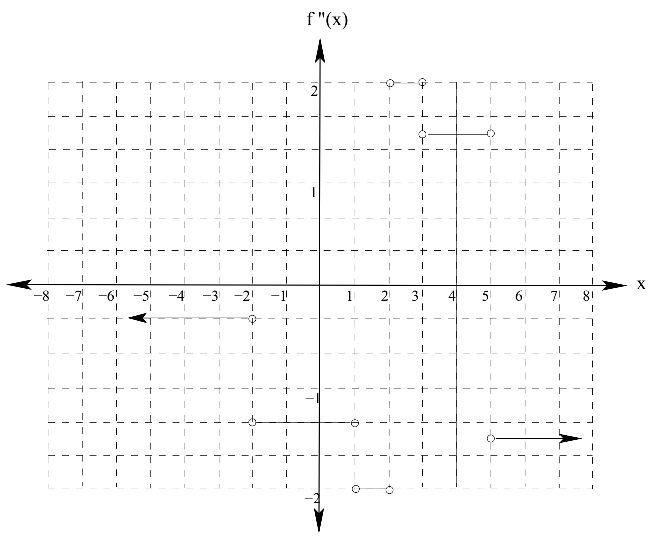
    (h) Sketch a possible graph for $f(x)$.



(i) Find the location of the absolute maximum on $[0,7]$, if one exists.
The absolute max is when $x = 7$

(j) Find the location of the absolute minimum on $[0,7]$, if one exists.
The absolute min is when $x = 2$ (this requires us to notice that the amount added to $f(x)$ as x goes from 0 to 1 is less than the amount subtracted from $f(x)$ as x goes from 1 to 3, so $f(0) > f(3)$.

(k) Sketch a possible graph of f''



25. Find the absolute maximum and the absolute minimum of each of the following functions on the given interval.

(a) $f(x) = x^2 - 10x + 12$ on the interval $[-1,7]$
First notice that f is continuous so the EVT applies. Next, $f'(x) = 2x - 10 = 0$ when $x = 5$
Now $f(-1) = 23$; $f(5) = -13$, and $f(7) = -9$, so the absolute max is: 23 and the absolute min is: -13

(b) $f(x) = x^2 - 6x + 7$ on the interval $[-2,2]$
First notice that f is continuous so the EVT applies. Next, $f'(x) = 2x - 6 = 0$ when $x = 3$, but this is not in our interval of interest.
Now $f(-2) = 23$ and $f(2) = -1$, so the absolute max is: 23 and the absolute min is: -1

(c) $f(x) = \dfrac{1}{3}x^3 - 2x^2 + 3x + 5$ on the interval $[0,4]$
First notice that f is continuous so the EVT applies. Next, $f'(x) = x^2 - 4x + 3 = 0$ when $x = 1$ or $x = 3$.
Now $f(0) = 5$; $f(1) = \frac{19}{3}$, $f(3) = 5$ and $f(4) = \frac{19}{3}$, so the absolute max is: $\frac{19}{3}$ and the absolute min is: 5.

(d) $f(x) = \dfrac{x+1}{2x-3}$ on the interval $[2,5]$

First notice that f is continuous on $[2,5]$ (the only value we need to worry about is $x = \frac{3}{2}$) so the EVT applies. Next, $f'(x) = \frac{(2x-3)-(x+1)2}{(2x-3)^2} = \frac{-5}{(2x-3)^2}$ which is never zero.

Now $f(2) = 3$, and $f(5) = \frac{6}{7}$, so the absolute max is: 3 and the absolute min is: $\frac{6}{7}$.

(e) $f(x) = \dfrac{x}{2x-1}$ on the interval $[0,4]$

Notice that f is discontinuous at $x = \frac{1}{2}$, so the EVT does not apply. In fact, the function has a vertical asymptote that attains values approaching ∞ on one side and $-\infty$ on the other side, the this function has no absolute max and no absolute min on this interval.

26. Sketch a graph which satisfies the following:

- domain is $\{x|x \neq -3\}$
- y-intercept is $(0,-1)$
- x-intercepts are $(-4,0)$, $(-1,0)$, and $(7,0)$
- the vertical asymptote is at $x = -3$
- $\lim_{x\to-\infty} f(x) = -2$
- $\lim_{x\to\infty} f(x) = 3$
- increasing on $(-\infty,-3)\cup(4,\infty)$
- **de**creasing on $(-3,4)$ (oops - typo on original handout!)
- local min at $(4,-5)$
- no local maxima
- concave up on $(-\infty,-3)\cup(-3,0)\cup(2,7)$
- concave down on $(0,2)\cup(7,\infty)$
- inflection points at $(0,-1),(2,-2),(7,0)$