40. Consider $f(x) = 2x^2 - 3$ in the inteval $[2, 5]$

(a) Find a summation formula that gives an estimate the definite integral of f on $[2, 5]$ using n equal width rectangles and using left hand endpoints to give the height of each rectangle. You do not have to evaluate the sum or find the exact area.

First notice that $\Delta x = \frac{5-2}{n} = \frac{3}{n}$.

Then $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(2 + (k-1)\Delta x)\Delta x = \lim_{n\to\infty} \sum_{k=1}^{n} [2(2 + (k-1)\frac{3}{n})^2 - 3]\frac{n}{3}$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[2\left(4 + \frac{4n(k-1)}{3} + \frac{n^2(k-1)^2}{9}\right) - 3\right]\frac{n}{3} = \lim_{n\to\infty} \sum_{k=1}^{n} \left[8 + \frac{8n(k-1)}{3} + \frac{2n^2k^2 - 4n^2k + 2n^2}{9} - 3\right]\frac{n}{3}$

(b) Find the norm of the partition $P : 2 < 3 < 3.5 < 4 < 4.5 < 5$

The norm is the width of the largest subinterval, or 1.

(c) Find the approximation of the definite integral of f on $[2, 5]$ using the Riemann sum for the partition P given in part (b).

Since we are using rectangles with left hand endpoints, the sum is given by:

$A \approx (1)(f(2)) + (.5)(f(3)) + (.5)(f(3.5)) + (.5)(f(4)) + (.5)(f(4.5)) = (5) + (7.5) + (10.75) + (14.5) + (18.75) = 56.5$

41. Assume f is continuous on $[-1, 4]$, $\int_{-1}^{1} f(x)\,dx = 5$, $\int_{1}^{4} f(x)\,dx = -1$, and $\int_{2}^{4} f(x)\,dx = 2$. Find:

(a) $\int_{1}^{-1} f(x)\,dx = -5$

(b) $\int_{-1}^{4} f(x)\,dx = 5 - 1 = 4$

(c) $\int_{1}^{2} f(x)\,dx = -1 - 2 = -3$

(d) $\int_{-1}^{2} f(x)\,dx = 5 - 3 = 2$

(e) Find the average value of f on $[-1, 1]$

The average of f on $[-1, 1]$ is given by: $\frac{1}{1-(-1)}\int_{-1}^{1} f(x)\,dx = \frac{1}{2}5 = \frac{5}{2}$.

42. Evaluate the following:

(a) $\int_{1}^{4} 3x^2 + \sqrt{x} + 2\,dx$

$= x^3 + \frac{2}{3}x^{\frac{3}{2}} + 2x\Big|_1^4 = (64 + \frac{16}{3} + 8) - (1 + \frac{2}{3} + 2) = \frac{221}{3} \approx 73.667$

(b) $\int_{0}^{1} x^2(x^3 + 5)^2\,dx$

Let $u = x^3 + 5$ Then $du = 3x^2\,dx$, or $\frac{1}{3}du = x^2\,dx$. Also, $0^3 + 5 = 5$ and $1^3 + 5 = 6$

This gives the integral: $\frac{1}{3}\int_{5}^{6} u^2\,du = \frac{1}{3}u^3\Big|_5^6 = \frac{216}{9} - \frac{125}{9} = \frac{91}{9} \approx 10.111$

(c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3(3x)\sin(3x)\,dx$

Let $u = \cos(3x)$ Then $du = -3\sin(3x)\,dx$, or $-\frac{1}{3}du = \sin(3x)\,dx$. Also, $\cos(\frac{3\pi}{6}) = 0$ and $\cos(\frac{3\pi}{2}) = 0$.

This gives the integral: $-\frac{1}{3}\int_{0}^{0} u^3\,du = 0$

(d) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin x\,dx$

Notice that $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin x\,dx = 0$ since $\sin x$ is odd.

43. Compute the following:

(a) $\dfrac{d}{dx}\left(\displaystyle\int_2^{x^2-1} \frac{t}{\sqrt{t-1}}\,dt\right)$ (b) $\displaystyle\int \frac{d}{dt}\left(\frac{t}{\sqrt{t-1}}\right)\,dt$

(a) Using the first part of the fundamental theorem of calculus, $\dfrac{d}{dx}\left(\displaystyle\int_2^{x^2-1} \frac{t}{\sqrt{t-1}}\,dt\right) = \dfrac{d}{dx}\left(F(x^2-1) - F(2)\right)$

$= \frac{x^2-1}{\sqrt{x^2-1-1}} \cdot 2x = \frac{2x^3-2x}{\sqrt{x^2-2}}$

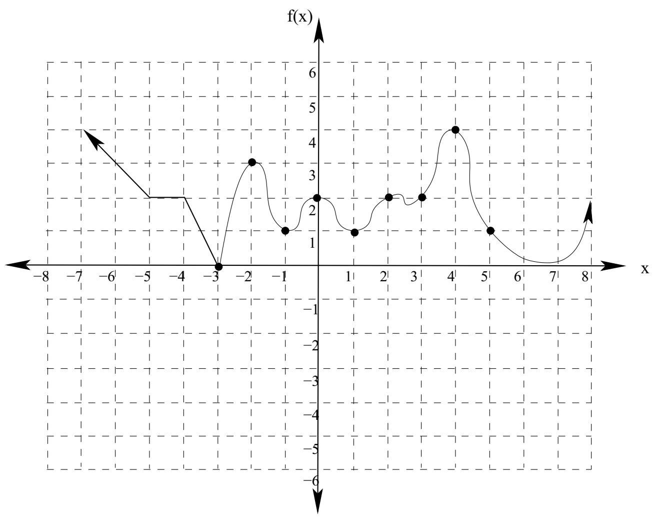
(b) Again applying the the fundamental theorem of calculus, $\displaystyle\int \frac{d}{dt}\left(\frac{t}{\sqrt{t-1}}\right)\,dt = \frac{t}{\sqrt{t-1}}$

(c) $\dfrac{d}{dx}\left(\displaystyle\int_2^{5} \frac{t}{\sqrt{t-1}}\,dt\right)$ (d) $\displaystyle\int_2^5 \left[\frac{d}{dt}\left(\frac{t}{\sqrt{t-1}}\right)\,dt\right]$

(c) Since $\displaystyle\int_2^5 \frac{t}{\sqrt{t-1}}\,dt$ is a constant, $\dfrac{d}{dx}\left(\displaystyle\int_2^{5} \frac{t}{\sqrt{t-1}}\,dt\right) = 0$

(d) $\displaystyle\int_2^5 \left[\frac{d}{dt}\left(\frac{t}{\sqrt{t-1}}\right)\,dt\right] = \frac{5}{\sqrt{5-1}} - \frac{2}{\sqrt{2-1}} = .5$

44. Given the following graph of $f(x)$:



(a) Approximate $\displaystyle\int_{-3}^{5} f(x)\,dx$ using 8 equal width rectangles with height given by the right hand endpoint of each rectangle.

(b) Use the Trapezoid Rule with $n = 4$ to approximate $\displaystyle\int_{-3}^{5} f(x)\,dx$.

45. (a) Use the Trapeziodal Rule with $n = 4$ to approximate $\displaystyle\int_0^{\pi} 3\sin x\;dx$

(b) Find the maximum possible error in your approximation from part (a).

(c) Find the minimum number of rectangles that should be used to guarantee an approximation of $\displaystyle\int_0^{\pi} 3\sin x\;dx$ to within 4 decimal places using the Trapeziodal Rule.

(d) Use the Fundamental Theorem of Calculus to find $\displaystyle\int_0^4 2x^3\;dx$ exactly. How far off was your estimate? How does the actual error compare to the maximum possible error?

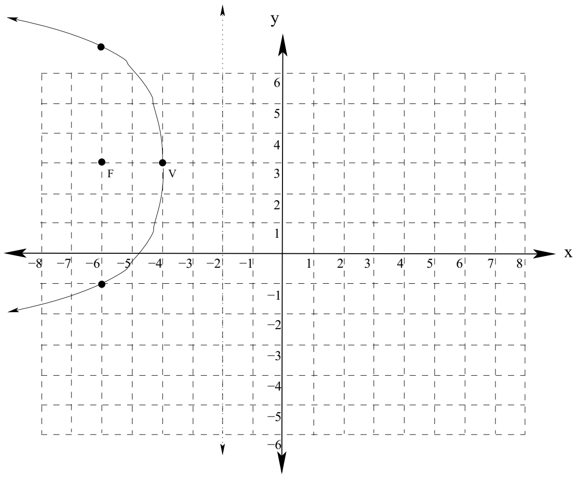
46. Graphs the following conic sections. Clearly label the main geometric features.

(a) $y^2 - 6y + 8x + 41 = 0$ (oops - slight typo!)

Then $y^2 - 6y = -8x - 41$, so $y^2 - 6y + 9 = -8x - 41 + 9 = -8x - 32$

Therefore $(y-3)^2 = -8(x+4) = 4(-2)(x+4)$. Thus we see that the vertex of this parabola is $V : (-4, 3)$. From the form of the equation, we see that this parabola opens left and $p = -2$, so the focus is $F : (-6, 3)$ and the directrix is $x = -2$

Finally, if we let $x = -6$ and solve using our equation, we see that the points $(-6, -1)$ and $(-6, 7)$ are on the parabola. This gives the graph:

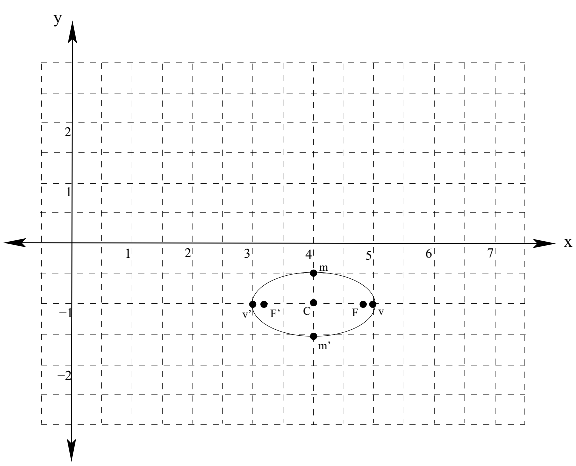


(b) $x^2 + 4y^2 - 8x + 8y + 19 = 0$

By completing the square we obtain: $x^2 - 8x + 16 + 4(y^2 + +1) = -19 + 16 + 4$

Therefore, $(x-4)^2 + 4(y+1)^2 = 1$, or $\frac{x-4)^2}{1} + \frac{(y+1)^2}{\frac{1}{4}} = 1$

Thus we have $C : (4, -1)$, $a = 1$ and $b = \frac{1}{2}$, so $c^2 = 1 - \frac{1}{4} = \frac{3}{4}$, so $c = \frac{\sqrt{3}}{2}$. Hence we have $V : (5, -1)$, $V' : (3, -1)$, $F \approx (3.134, -1)$, $F' : (4.866, -1)$, $M : (4, -.5)$, and $M' : (4, -1.5)$. This gives the graph:

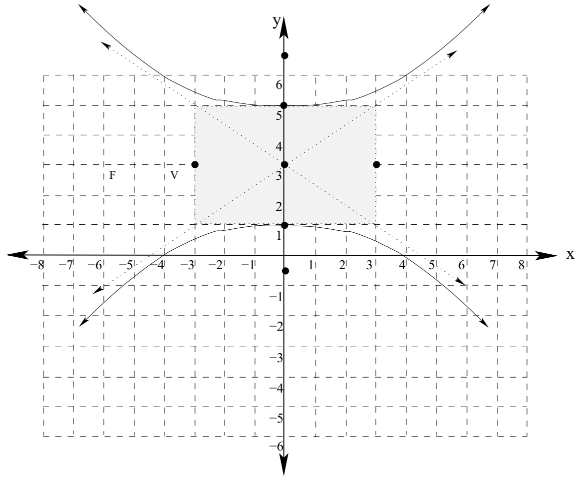


(c) $9y^2 - 4x^2 - 54y + 45 = 0$

By completing the square we obtain: $9(y^2 - 54y + 9) - 4(x^2) = -45 + 81$

Therefore, $9(y-3)^2 - 4x^2 = 36$, or $\frac{y-3)^2}{4} - \frac{(x^2}{9} = 1$

Thus we have $C : (0, 3)$, $a = 2$ and $b = 3$, so $c^2 = 4 + 9 = 13$, so $c = \sqrt{13}$. Hence we have $V : (0, 5)$, $V' : (0, 1)$, $F \approx (0, 6.6056)$, $F' : (0, -.6056)$, $M : (-3, 3)$, and $M' : (3, 3)$. This gives the graph:



47. Find an equation for the conic section with the given features:

(a) A parabola with vertex $(-3, 5)$, directrix parallel to the x-axis, and passing through the point $(5, 9)$

Since the directrox is vertical, we have a parabola that opens left/right, so it will have an equation of the form: $(y - k)^2 = 4p(x - h)$. We know that the vertex is $(-3, 5)$ and that $(5, 9)$ is on the parabola, so, substituting these values:

$(9 - 5)^2 = 4p(5 - (-3))$, or $16 = 4p(8)$, so $p = \frac{1}{2}$

Therefore the equation is: $(y - 5)^2 = 2(x + 3)$

(b) An ellipse with center $(0, 0)$ passing through the points $(2, 3)$ and $(6, 1)$

Since we are looking at an ellipse centered at the origin, is has an equation of the form:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Substituting the known points gives:

$\frac{4}{a^2} + \frac{9}{b^2} = 1$ and $\frac{36}{a^2} + \frac{1}{b^2} = 1$

Therefore, multiplying both sides of these by a^2b^2 gives $4b^2 + 9a^2 = a^2b^2$ and $36b^2 + a^2 = a^2b^2$. Thus $4b^2 + 9a^2 = 36b^2 + a^2$, or $8a^2 = 32b^2$, hence $a^2 = 4b^2$, or $a = 2b$ (since both are known to be positive).

Substituting this back into one of our previous equations, we see that $\frac{4}{4b^2} + \frac{9}{b^2} = 1$, or, solving this, $b^2 = 10$, and $a^2 = 40$

Hence the equation is: $\frac{x^2}{40} + \frac{y^2}{10} = 1$

(c) A hyperbola with foci $(0, \pm 3)$ and vertices $(0, \pm 2)$

This is clearly a hyperbola that open up/down and it is symmetric with respect to the both coordinate axes, so the center must be $(0, 0)$. Also, we see that $c = 3$ and $a = 2$. Therefore, $b^2 = c^2 - a^2 = 9 - 4 = 5$

Thus the equation must be: $\frac{y^2}{4} - \frac{x^2}{5} = 1$