Math 261, Section 2
Quiz 2 - Solutions
9/21/2007

Name:______________________________

1. (5 points) Use the formal limit definition to find the derivative of the function $f(x) = \dfrac{3}{x}$

   (no credit will be given for finding $f'(x)$ using a differentiation formula)

   $$f'(x) = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \lim_{h\to 0} \frac{\frac{3\cdot x}{x(x+h)} - \frac{3(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{3x-3x-3h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-3h}{x(x+h)}\frac{1}{h} = \lim_{h\to 0} \frac{-3}{x(x+h)} = \frac{-3}{x^2}$$

2. (3 points) Find the equation for the tangent line to $f(x) = \dfrac{3}{x}$ when $x = 1$

   Since $f'(x) = \frac{-3}{x^2}$, the slope of the tangent line when $x = 1$ is $f'(1) = \frac{-3}{1^2} = -3$

   The point of tangency is: $P = (1, f(1)) = (1, \frac{3}{1}) = (1, 3)$.

   Therefore, using the point-slope formula, the equation for the tangent line is given by:

   $y - 3 = -3(x - 1) = -3x + 3$, or $y = -3x + 6$.

3. (2 points) Find all points on the graph of $f(x) = \dfrac{3}{x}$ at which the slope of the tangent line is -27.

   If $m = -27$, then $f'(x) = \frac{-3}{x^2} = -27$. Thus $-3 = -27x^2$, or $x^2 = \frac{-3}{-27} = \frac{1}{9}$. So $x = \pm\frac{1}{3}$

   Finding the corresponding y - coordinates, $f(\frac{1}{3}) = \frac{3}{\frac{1}{3}} = 9$, and $f(-\frac{1}{3}) = \frac{3}{-\frac{1}{3}} = -9$.

   Hence the points on the graph of f at which the slope of the tangent line is -27 are $(\frac{1}{3}, 9)$ and $(-\frac{1}{3}, -9)$.