

1. Find each limit, (if it exists).

(a)  $\lim_{x \rightarrow 0^+} \frac{\ln x}{\csc x}$

(b)  $\lim_{x \rightarrow 1} \frac{2x^2 - (3x + 1)\sqrt{x} + 2}{x - 1}$

(c)  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x + 1}}{\sqrt{x + 1}}$

(d)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

(e)  $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$

(f)  $\lim_{x \rightarrow \infty} \frac{2x^2 - 1}{5x^2 + 3x}$

(g)  $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 3}$

$$(h) \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

$$(i) \lim_{x \rightarrow 0} x \cot x$$

$$(j) \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$

$$(k) \lim_{x \rightarrow 0^+} (1 + ax)^{\frac{b}{x}}$$

$$(l) \lim_{x \rightarrow 0^+} x^x$$

2. Find a value of  $c$  that makes the function

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

continuous at  $x = 0$

3. (a) The **sine integral**  $Si(x) = \int_0^x \frac{\sin u}{u} du$  is a useful function in applied mathematics.

Find  $\lim_{x \rightarrow 0} \frac{Si(x)}{x}$

- (b) The **Fresnel cosine integral**  $C(x) = \int_0^x \cos(u^2) du$  is used in the analysis of the diffraction of light.

Find:  $\lim_{x \rightarrow 0} \frac{C(x) - x}{x^5}$

4. If air resistance is disregarded, the acceleration of a falling object is given by  $a(t) = g$ , where  $g$  is a gravitational constant, so the velocity of an object  $t$  seconds after being released is given by:  $v(t) = gt$ .

If we assume air resistance is proportional to the velocity  $v$ , it can be shown that the velocity of an object  $t$  seconds after being released is given by:  $v(t) = \frac{mg}{k} (1 - e^{-kt/m})$ , where  $m$  is the mass of the object and  $k$  is a constant air-resistance coefficient. In the following problems, we will explore the consequences of taking air resistance into account when studying the velocity,  $v(t)$  of a falling object.

(a) Find  $\lim_{k \rightarrow 0^+} v(t)$

[Consequently,  $k = 0$  corresponds to the case of no air resistance.]

(b) Find  $\lim_{k \rightarrow \infty} v(t)$

[Consequently, a body's velocity tends to a finite limit, its *terminal velocity*, after it has fallen a sufficiently long time.]

(c) Find  $\lim_{m \rightarrow 0^+} v(t)$

[Consequently, a light "feathery" body falls very slowly through the air.]

(d) Find  $\lim_{m \rightarrow \infty} v(t)$

[Consequently, a heavy body tends to fall much as it would with no air resistance.]