

1. Determine whether the following improper integrals converge or diverge. For those that do converge, find their value.

(a)
$$\int_2^{\infty} \frac{1}{x^2} dx$$

(d)
$$\int_0^{\infty} \frac{x}{e^x} dx$$

(b)
$$\int_2^{\infty} \frac{1}{\sqrt{x}} dx$$

(e)
$$\int_1^5 \frac{1}{\sqrt{x-1}} dx$$

(c)
$$\int_2^{\infty} \frac{1}{x \ln x} dx$$

(f)
$$\int_{-1}^0 \frac{1}{\sqrt[5]{x}} dx$$

(g) $\int_1^5 \frac{1}{x^2 - 16} dx$

(h) $\int_0^\infty \frac{1}{x^2} dx$

2. Use comparisons to determine whether the following improper integrals converge or diverge:

(a) $\int_1^\infty \frac{2 + \cos x}{x^2} dx$

(b) $\int_1^\infty \frac{1}{\sqrt[3]{x^3 - x}} dx$

3. Use comparisons to determine whether the following improper integrals converge or diverge:

(a) $\int_1^{\infty} \frac{1}{x^{1.1} + x + 2} dx$

(b) $\int_1^{\infty} \frac{1}{x^{0.9} + x + 2} dx$

4. Show that $\int_{-\infty}^{\infty} \frac{1+x}{1+x^2} dx$ diverges, but that $\lim_{t \rightarrow \infty} \int_{-t}^t \frac{1+x}{1+x^2} dx = \pi$

5. A mechanic decides to make a funnel by rotating the curve $y = \frac{1}{x}$ from $x = 1$ to $x = H$ about the x -axis.

(a) Find the volume of the funnel, in terms of H .

(b) Set up (but DO NOT evaluate) an integral that represents the surface area of the funnel in terms of H .

(c) If we let $H \rightarrow \infty$, find the volume of the resulting “infinitely long” funnel (often called “Gabriel’s Horn”)

(d) Use a comparison to show that if we let $H \rightarrow \infty$, the resulting “infinitely long” funnel has infinite surface area.