1. Find all values for $p$ for which the integral $\int_1^\infty \frac{1}{x^p}\,dx$ converges. Note: The parameter $p$ can be any real number: positive or negative, rational or irrational. Also be careful to check for special cases.

2. (Modified from the 2007 AP Calculus AB exam) Let $R$ be the region in the first and second quadrants that is bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$. Let $S$ be the region in the first quadrant bounded above by $y = \frac{20}{1+x^2}$ and below by the $x$-axis. (This problem continues on the following pages.)

   (a) Evaluate $\int \frac{1}{(1+x^2)^2}\,dx$

(b) Evaluate $\int_0^3 \frac{1}{(1+x^2)^2}\,dx$.

(c) Evaluate $\int_0^\infty \frac{1}{(1+x^2)^2}\,dx$.

(d) Find the volume of the solid generated when $R$ is rotated about the $x$-axis.

(e) Find the volume of the solid generated when $R$ is rotated about the $y$-axis.

(f) Find the volume of the solid generated when $S$ is rotated about the $x$-axis.

(g) Find the volume of the solid generated when $S$ is rotated about the $y$-axis.

(h) Setup, but do not evaluate, an integral expression that gives the length of the boundary curve for the region $R$.

(i) Setup, but do not evaluate, an integral expression that gives the surface area of the solid that is formed when the region $R$ is rotated about the line $y = 2$.

(j) The region $R$ is the base of a solid. For this solid, the cross-sections perpendicular to the $x$-axis are semicircles. Find the volume of this solid.