1. In each of the following problems, a formula for the nth term a_n of a sequence $\{a_n\}$ is given. Find the first *four* terms: a_1, a_2, a_3, and a_4 of each sequence and find $\lim_{x \to \infty} a_n$ if it exists.

   (a) $a_n = \dfrac{3}{n!}$

   (b) $a_n = \dfrac{4 - n^2}{3 + n^2}$

   (c) $a_n = (-1)^n \dfrac{n}{n+1}$

   (d) $a_n = 7$

2. In each of the following problems, a sequence $\{a_n\}$ is defined recursively. Find the first *five* terms: a_1, a_2, a_3, a_4 and a_5 of each sequence. Then, using a calculator is necessary, give an educated guess for $\lim\limits_{x\to\infty} a_n$, if it exists.

   (a) $a_1 = 1 \quad a_{n+1} = a_n + \frac{1}{2^n}$ for $n \geq 1$

   (b) $a_1 = \sqrt{6} \quad a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$

   (c) $a_1 = 1 \;\; a_2 = 1 \quad a_{n+2} = a_{n+1} + a_n$ for $n \geq 1$

3. Determine whether each sequence converges or diverges. For those that converge, find the limit.

(a) $\left\{ \dfrac{1 - 5n^4}{n^4 + 7n^3} \right\}$

(b) $\left\{ \left(1 + \dfrac{7}{n}\right)^n \right\}$

(c) $\left\{ \dfrac{2^n + 5}{3^n} \right\}$

(d) $\left\{\sec^{-1} n\right\}$

(e) $\left\{\dfrac{\sin n}{n^2}\right\}$

(f) $\{\ln(2n+1) - \ln n\}$

(g) $\left\{\dfrac{n!}{n^n}\right\}$ [Hint: Compare with $\frac{1}{n}$].

4. This problem will investigate the sequence $\{a_n\}$ that is defined recursively by:
$a_1 = \sqrt{6} \quad a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$

   (a) Is $a_1 < 3$?

   (b) Show that if we assume that a particular term a_n satisfies the inequality $a_n < 3$, then it follows that $a_{n+1} < 3$.

   Note: (a) and (b) together imply that every term in this sequence is less than 3 [this type of proof is called a "proof by mathematical induction"].

   (c) Since all terms of the sequence $\{a_n\}$ are less than 3, this sequence is said to be ____________.

   (d) Show that $a_{n+1} > a_n$ for all $n \geq 1$. [Hint: Part of your proof should include calculating $(a_{n+1})^2 - (a_n)^2$ and showing that $(a_{n+1})^2 - (a_n)^2 > 0$]

   (e) By (d), $a_1 \leq a_2 \leq \cdots \leq a_n \leq \cdots$. Hence the sequence $\{a_n\}$ is said to be ____________.

   (f) From (c) and (e), we can conclude that the sequence $\{a_n\}$ ____________.

   (g) Let $\lim_{n\to\infty} a_n = A$.

      i. Then $\lim_{n\to\infty} a_{n+1} = A$. Why?

      ii. Use the fact that $\lim_{n\to\infty} a_n = A$ to write the $\lim_{n\to\infty} \sqrt{6 + a_n}$ in terms of A.

      iii. But, $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{6 + a_n}$. Why?

      iv. Use (i), (ii), and (iii) to find A.

   Hence, the work done in this problem proves that the limit of $\{a_n\}$ converges to ____________.

5. The so-called **hailstone** sequence is defined by:

$$a_{n+1} = \begin{cases} 3a_n + 1 & \text{if } a_n \text{ is odd} \\ \frac{1}{2}a_n & \text{if } a_n \text{ is even} \end{cases}$$

(a) Let $a_1 = 17$. Write down the terms of the sequence until you reach 1.

(b) Let $a_1 = 30$. Write down the terms of the sequence until you reach 1.

(c) Start with some other positive integer value for a_1 and write down the terms of the sequence until you reach 1.

(d) Start with yet another positive integer value for a_1 and write down the terms of the sequence until you reach 1.

(e) Either show that you *always* reach 1 no matter which positive integer value for a_1 you start with *or* find an example of a sequence that you can prove never reaches 1.
(Attach additional work if needed)
[Note: If you are successful in solving this problem, you will be the first person in history to have solved this problem. No solution is known.]