1. Let $f(x) = -\ln(1-x)$.

   (a) Find $f'(x), f''(x), f'''(x), f^{(4)}(x)$, and $f^{(n)}(x)$

   (b) Find $f'(0), f''(0), f'''(0), f^{(4)}(0)$, and $f^{(n)}(0)$

   (c) Find the nth-degree Maclaurin polynomial of f

   (d) Find the Maclaurin series for $f(x)$. (Do not verify that $\lim_{n\to\infty} R_n(x) = 0$

   (e) Find the interval of convergence for this Maclaurin series.

2. Find the first five terms of the Taylor series for the following functions at a given value c:

   (a) $f(x) = \cos x$ at $c = \frac{\pi}{3}$

   (b) $f(x) = \sqrt{x}$ at $c = 4$

   (c) $f(x) = \tan x$ at $c = \frac{\pi}{4}$

3. Find the Maclaurin series for $f(x) = 6x^4 - 2x^3 + 4x^2 + x + 7$. Show all work!

4. Use a Maclaurin series to approximate the following improper integral to four decimal places:
$$\int_0^1 \frac{\sin x}{x}\, dx$$

5. (a) Use the Maclaurin series for $\arctan x$ with $x = 1$ to represent π as the sum of an infinite series.

(b) What accuracy is obtained by using the first five terms of the series to approximate π?

(c) Approximately how many terms of the series are required to obtain four decimal places of accuracy?

6. Find the sum of each of the following infinite series. Give exact answers.

(a) $1 - \dfrac{1}{2!} + \dfrac{1}{4!} - \dfrac{1}{6!} + \cdots + (-1)^n \dfrac{1}{(2n)!} + \cdots$

(b) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots + (-1)^n \dfrac{1}{n+1} + \cdots$

(c) $\dfrac{\pi}{6} - \dfrac{\pi^3}{6^3 \cdot 3!} + \dfrac{\pi^5}{6^5 \cdot 5!} - \cdots + (-1)^n \dfrac{\pi^{2n+1}}{6^{2n+1}(2n+1)!} + \cdots$

7. Find Taylor's Formula with remainder for the given $f(x)$, c, and n.

   (a) $f(x) = \cos x$, $c = \dfrac{\pi}{4}$, $n = 4$

   (b) $f(x) = \dfrac{1}{(x-3)^2}$, $c = 4$, $n = 5$

   (c) $f(x) = e^{-x^2}$, $c = 0$, $n = 3$

   (d) $f(x) = 3x^5 - 2x^3 + 5x^2 - 7$, $c = 0$, $n = 5$

8. If we used problem 7(a) to approximate $\cos 47°$, what decimal place accuracy could we achieve?

9. (a) Use problem 7(b) to approximate $\dfrac{1}{(.9)^2}$

(b) Use the Taylor remainder in problem 7(b) to estimate the error in this approximation.

10. Determine the number of decimal places of accuracy that the given approximation formula yields for $|x| \leq 0.1$.

(a) $e^x \approx 1 + x + \dfrac{1}{2}x^2 + \dfrac{1}{6}x^3 + \dfrac{1}{24}x^4$

(b) $\sin x \approx x - \dfrac{1}{6}x^3 + \dfrac{1}{120}x^5$

(c) $\ln(1 + x) \approx x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 - \dfrac{1}{4}x^4$