

1. Fill in each blank with the best word or phrase:

(a) A sequence $\{a_n\}$ converges to L if _____.

(b) $\lim_{n \rightarrow \infty} r^n = 0$ if _____.

$\lim_{n \rightarrow \infty} |r^n| = \infty$ if _____.

(c) Let $a \neq 0$. The series $a + ar + ar^2 + \dots + ar^{n-1} + \dots$

is called a _____ series.

it converges and has the sum $S =$ _____ if _____.

it diverges if _____.

(d) If $S_n = \frac{3n+1}{2n-5}$ then $\sum_{n=1}^{\infty} a_n =$ _____.

(e) If $S_n = \frac{3n+1}{2n-5}$ then $a_5 =$ _____.

(f) If $a_n = \frac{3n+1}{2n-5}$ then $\sum_{n=1}^{\infty} a_n =$ _____.

(g) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ _____.

(h) If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ _____.

(i) If $\sum_{n=1}^{\infty} a_n$ is a convergent series and $\sum_{n=1}^{\infty} b_n$ is a divergent series

then $\sum_{n=1}^{\infty} a_n + b_n$ is a _____ series.

(j) The series $\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ is called a _____ series.

It converges if _____ and diverges if _____.

2. Determine whether the following sequences converge or diverge. For those that converge, find the limit.

(a) $\left\{ 1 + \frac{(-1)^n}{n} \right\}$

(b) $\left\{ \frac{4^n - 7}{9^n} \right\}$

3. Find the sum of each series:

(a) $\sum_{n=2}^{\infty} e^{-n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{(2n-3)(2n-1)}$

4. Determine whether the following series converge or diverge. Make sure to show all work leading to your conclusion.

(a)
$$\sum_{n=1}^{\infty} \frac{3n + 5}{n^2 + 7}$$

(b)
$$\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

(c)
$$\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$$

5. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Make sure to show all work leading to your conclusion.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{3 + \ln n}$$

(c)
$$\sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{n^3+1}$$

6. For each of the following power series, find the interval of convergence and the radius of convergence:

(a) $\sum_{n=0}^{\infty} \frac{1}{n3^n} (x+4)^n$

Interval of Convergence: _____

Radius of Convergence: _____

(b) $\sum_{n=0}^{\infty} \frac{1}{n^n} x^n$

Interval of Convergence: _____

Radius of Convergence: _____

7. Find a power series in x that has the given function as its sum. Also find the interval of convergence.

(a) $\frac{1}{1+x^3}$

(b) $\sin \frac{2x}{3}$

8. Find the first four terms of the Taylor series for the function $f(x) = \frac{1}{1-x}$ centered at $c = 2$.

9. Find Taylor's formula with remainder for $f(x) = \ln(\cos x)$ with $c = \frac{\pi}{6}$ and $n = 3$

10. Approximate the integral $\int_0^1 x \sin(x^3) dx$ to six decimal places.