You MUST use good notation and show appropriate work.

Math 102

(Section 2.5B Supplemental)

Name ___________________________________

2.5B Supplemental

As arguments become more complicated it is often impractical to check an argument for validity by using truth tables. Instead of using truth tables to test whether a conclusion is valid, mathematicians often use a formal proof. We will restrict our formal proofs to those that are called direct proofs. By a direct proof we will mean a sequence of true statements such that:

i) the statement is a premise which is given to be true

or

ii) the statement is the conclusion of a valid argument from a given list of valid arguments such that its premises are from preceding statements in the sequence

or

iii) the statement is a logically equivalent statement to a preceding statement in the sequence

and

iv) the last statement in the sequence is the statement (conclusion) to be proved.

Some useful valid arguments (forms)

1) Law of Detachment $p \to q, \quad p, \quad \therefore q$

2) Law of Contraposition $p \to q, \ \sim q, \ \therefore \ \sim p$

3) Law of Syllogism $p \to q, \quad q \to r, \ \therefore p \to r$

4) Disjunctive Syllogism $p \vee q, \quad \sim p, \quad \therefore \ q$

5) Simplification $p \wedge q, \ \therefore p$ (or one could conclude q)

6) Addition: $p, \ \therefore \ p \vee q$

Some logically equivalent statements (forms)

1) Double negation: p and $\sim(\sim p)$

2) Contraposition: $p \to q$ and $\sim q \to \sim p$

3) De Morgan's laws: *i*) $\sim(p \wedge q)$ and $\sim p \vee \sim q$

ii) $\sim(p \vee q)$ and $\sim p \wedge \sim q$

4) Conditional to Disjunction: $p \to q$ and $\sim p \vee q$

5) Commutativity: $p \wedge q$ and $q \wedge p$, also $p \vee q$ and $q \vee p$

(over)

1. Examples of formal proofs.

a) Construct a “formal proof" in order to establish that the following argument is valid:

$$\begin{array}{r} (q \vee r) \to p \\ \sim p \\ s \to r \\ \hline \therefore \ \sim s \end{array}$$

Solution:

	Statement	Reason
1.	$(q \vee r) \to p$	premise
2.	$\sim p$	premise
3.	$s \to r$	premise
4.	$\sim (q \vee r)$	1, 2 law of contraposition
5.	$\sim q \wedge \sim r$	4, De Morgan
6.	$\sim r$	5 simplification
7.	$\therefore \sim s$	3, 6 law of contraposition

b) Following is a possible list of statements in a formal proof of the argument form:

$$\begin{array}{l} p \to q \\ \sim q \\ p \vee s \\ \hline \therefore \ s \end{array}$$

Provide reasons for each statement.

	Statement	Reason
1.	$p \to q$	
2.	$p \vee s$	
3.	$\sim p \to s$	
4.	$\sim q \to \sim p$	
5.	$\sim q \to s$	
6.	~q	
7.	$\therefore s$	

2. Some practice examples

a) $$\begin{array}{l} p \to q \\ r \to \sim q \\ \hline \therefore p \to \sim r \end{array}$$

b) $$\begin{array}{l} \sim s \\ p \to r \\ (p \to q) \to s \\ \hline \therefore \ r \end{array}$$

c) $$\begin{array}{r} \sim p \to (q \vee p) \\ \sim p \\ \hline \therefore q \end{array}$$