

Math 102

Exam 1: Additional Practice Problem Solutions

1. List the 7 problem solving strategies from section 1.1 in your book.

- (a) Draw Pictures
- (b) Choose Good Names for Unknowns
- (c) Be Systematic
- (d) Look for Patterns
- (e) Try a Simpler Version of the Problem
- (f) Guided Guessing (or - Guessing is OK)
- (g) Convert a New Problem in to an Older One

2. Determine whether each statement below is true or false.

If it is true, explain how you know that it is true, and if it is false, give an example that shows it must be false.

(a) $\frac{a+b}{a+c} = \frac{b}{c}$

False. For example, take $a = 2$, $b = 3$ and $c = 5$. Then $\frac{a+b}{a+c} = \frac{2+3}{2+5} = \frac{5}{7}$, while $\frac{b}{c} = \frac{3}{5}$, and $\frac{5}{7} \neq \frac{3}{5}$.

(b) 150 is an overestimate of 13.21×9.17

True. Round 13.21 up to 15 and 9.17 up to 10, and we get the overestimate $15 \times 10 = 150$.

(c) \$ 9,994.99 rounded to the bearest dollar is greater than \$ 9,994.99 rounded to the nearest 10 dollars.

True. Notice that \$ 9,994.99 rounded to the nearest dollar is \$ 9,995.00, while \$ 9,994.99 rounded to the nearest 10 dollars is \$ 9,990.00, and \$ 9,995.00 $\dot{>}$ \$ 9,990.00.

(d) If $A \cap B = \emptyset$, then A is empty or B is empty.

False. $A \cap B = \emptyset$ only means that A and B are disjoint, that is, they have no elements in common. Both A and B can have lots of elements in them. For example, suppose $A = \{x \mid x \text{ is an odd natural number}\}$, and $A = \{x \mid x \text{ is an even natural number}\}$. Then both A and B are infinite, but $A \cap B = \emptyset$.

(e) If A and B are equal as sets, then A and B are also equivalent as sets.

True. If $A = B$, then A and B have exactly the some elements, so they also must have the same number of elements, which is what it means for to sets to be equivalent.

3. Suppose you and a friend go out to dinner at a nice restaurant, and your bill for the meal is \$37.43. Give both an over-estimate and an under-estimate for a 15% tip for your server based on the amount of your total bill.

An overestimate for a 15% tip could be found by rounding the bill up to \$40.00, then 15% of \$40.00 is \$6.00

An underestimate for a 15% tip could be found by rounding the bill down to \$35.00, then 15% of \$35.00 is \$5.25

4. (a) Use set notation to list all the elements of the set:

$$A = \{x \mid x \text{ is an odd whole number less than } 21\}$$

$$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

(b) Use set-builder notation to describe the set $\{-3, -2, -1, 0, 1, 2, 3\}$

$$\{x \mid x \text{ is an integer that is greater than } -4 \text{ and less than } 4\}$$

5. Given that $A = \{ x \mid x \text{ is a letter in the word } \textit{banana} \}$, $B = \{ x \mid x \text{ is a letter in the word } \textit{bandana} \}$, $C = \{ x \mid x \text{ is a letter in the word } \textit{band} \}$, and $D = \{ \emptyset \}$, indicate whether the following are True or False (you do NOT need to justify your answers)

- (a) $\{a\} \in A$ False: $\{a\}$ is a set not an element
 (b) $d \in A$ False
 (c) $\emptyset \subseteq A$ True: \emptyset is a subset of **any** set.
 (d) $A \subseteq B$ True
 (e) $B \subset C$ False
 (f) $B = C$ True.
 (g) $D \subset B$ False: \emptyset is not an element of B
 (h) $\emptyset \in D$ True.

6. Let $U = \{ x \mid x \text{ is a whole number less than } 12 \}$; $A = \{0, 2, 4, 6, 8\}$; $B = \{1, 2, 3, 4, 5\}$; $C = \{6, 7, 8, 9, 10\}$

- (a) Write U in roster form
 $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
 (b) $A \cap B = \{2, 4\}$
 (c) $A \cup C = \{0, 2, 4, 6, 7, 8, 9, 10\}$
 (d) $A' = \{1, 3, 5, 7, 9, 10, 11\}$
 (e) $B \cap C = \emptyset$
 (f) $B - A = \{1, 3, 5\}$
 (g) $n(A \cap C) = 2$
 (h) $n(C' \cap B) = 5$

Note: For part (g), notice that $A \cap C = \{6, 8\}$, and for part (h), notice that $C' \cap B = \{1, 2, 3, 4, 5\}$.

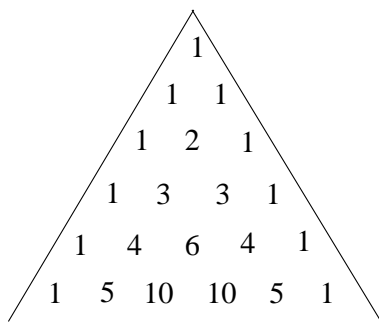
7. (a) List all the subsets of the set $\{a, b, c\}$

- \emptyset
 $\{a\}$; $\{b\}$; $\{c\}$
 $\{a, b\}$; $\{a, c\}$; $\{b, c\}$
 $\{a, b, c\}$

(b) How many subsets does the set $\{a, b, c, d, e\}$ have?

Since $\{a, b, c, d, e\}$ has 5 elements, it has $2^5 = 32$ different subsets

(c) Write out the first 6 rows of Pascal's Triangle

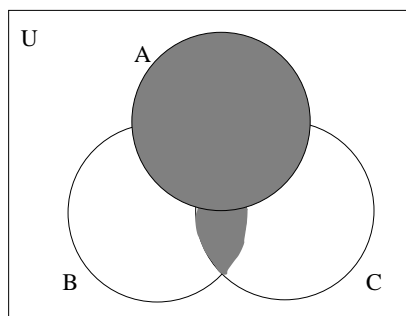
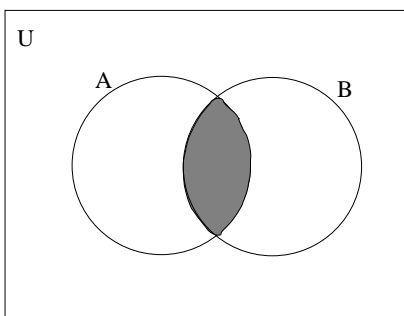
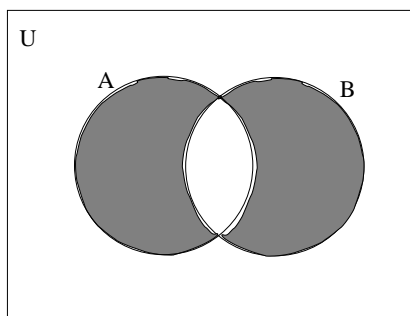


(d) Use Pascal's Triangle to find the number of 3 element subsets of $\{a, b, c, d, e\}$

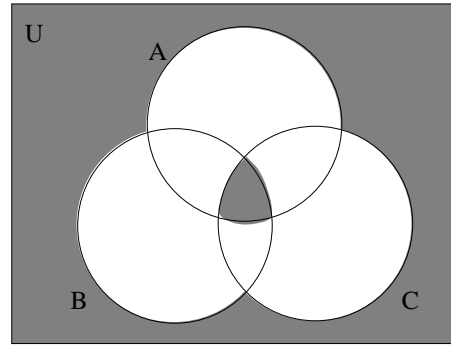
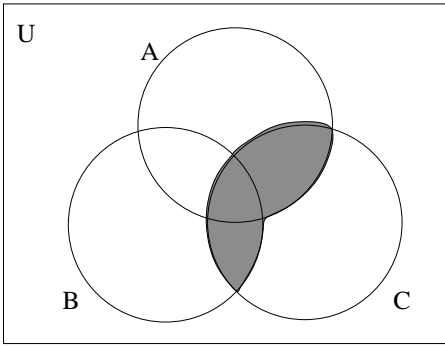
Since $\{a, b, c, d, e\}$ has 5 elements in it, we look at the 5th row in the triangle above. Since the 4th entry in this row corresponds to the number of subsets of size 3, there are 10 three element subsets of the set $\{a, b, c, d, e\}$

8. Illustrate the following by shading the appropriate regions of the given Venn diagrams:

- (a) $(A - B) \cup (B - A)$ (b) $A - B'$ (c) $A \cup (B \cap C)$



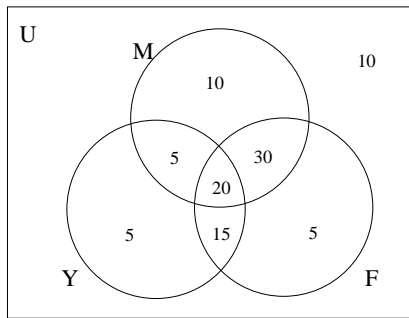
9. Use set notation to describe the shaded regions in each Venn diagram given below:



(a) $(A \cap C) \cup (B \cap C)$ or $C \cap (A \cup B)$

(b) $(A \cap B \cap C) \cup (A \cup B \cup C)'$ or $(A \cap B \cap C) \cup (A' \cap B' \cap C')$

10. A survey of 100 college students asked about the websites they visited that week among the 3 choices: Facebook, Myspace, and YouTube. Suppose the survey found that 70 visited Facebook, 35 visited YouTube and Facebook, 20 visited all three, 50 visited both Myspace and Facebook, 40 visited Myspace but not YouTube, 85 visited Facebook or Myspace, and 10 visited none of them.



(a) How many visited Myspace but not Facebook?

15

(b) How many visited Myspace and YouTube?

25

(c) How many visited Facebook or YouTube?

80

(d) How many visited Myspace?

65

(e) How many visited only YouTube?

5

11. For each of the following, state whether the situation is an example of inductive or deductive reasoning:

(a) You notice that your houseplants seem to grow better if you water them in the morning rather than in the evening, so you decide to start watering them every morning right before you leave to go to school.

Inductive Reasoning

(b) After hearing a debate on the radio, you decide to construct a truth table in order to determine whether or not the logical argument given by one of the participants is valid.

Deductive Reasoning

(c) The last couple of times you have gone to the grocery store on Friday afternoon, you noticed that they were giving away free samples, so you decide to start doing your grocery shopping on Friday afternoon every week.

Inductive Reasoning

13. Determine whether or not each of the following are statements:

12. Use inductive reasoning to predict the next two terms in each of the following sequences:

(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, \frac{5}{6}, \frac{6}{7}$

(b) 1, 2, 4, 7, 11, 16, ... **22, 29**

(c) 2, 5, 7, 12, 19, 31, ... **50, 81**

(a) I went to the store last night.

Statement

(b) Did you remember to get milk at the store?

Not a statement

(c) I forgot to pick up a gallon of milk at the store.

Statement

14. Negate each of the following statements, then rewrite them as English sentences:

(a) All bees are busy.

It is not the case that all bees are busy.

Some bees are not busy.

(b) Some things are better left unsaid.

It is not the case that some things are better left unsaid.

All things should be said.

(c) I got up early on Saturday and went to the gym.

It is not the case that I got up early on Saturday and went to the gym.

I did not get up early, or I did not go to the gym.

15. Given p : I studied for this exam, q : I got a good grade on this exam, r : I understand truth tables, and s : I am not good at doing proofs, translate the following statements into words:

(a) $p \wedge (\sim s) \rightarrow q$

If I studied for this exam and I am good at doing proofs then I got a good grade on this exam.

(b) $(\sim p \vee s) \rightarrow \sim q$

If I did not study for this exam or am not good at doing proofs then I did not get a good grade on this exam.