Math 102
Project 2 Handout
Due: 10/24/2008

Instructions: This project is designed to give you an opportunity to explore some additional concepts from mathematical logic. Complete as much of this project as you can by the due date (Friday February 29th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included as well. If you consult any references (books or on line material), cite the relevant sources either in footnotes, or at the end of your project.

1. (7 points) Given the argument:

   $s \rightarrow t$
   $p \vee q$
   $p \rightarrow \sim t$
   s

   $\therefore q$

   Write a two column proof that demonstrates that this argument is valid.

2. (3 points each) The following logic puzzles are based on the work of logician Raymond Smullyan. Suppose an alien planet has two types of inhabitants: *freeps*, who always tell the truth and plan to help humanity, and *floops*, who always lie and are bent on destroying humanity. Other than this difference, they look and act identically. Each part below describes an encounter with two inhabitants from this alien planet. Each part should be viewed as independent from the other parts – only use the basic rules and the information given in that part. Your job is to use the information determine the identity of the inhabitants mentioned in each part. That is, to determine whether each inhabitant mentioned is a freep or a floop. You must write out a proof in paragraph form which proves the identity of each alien creature in each problem.

   (a) $\mathbb{A}$ says: "$\mathbb{B}$ is a floop". $\mathbb{B}$ says: "Both of us are freeps".

   (b) $\mathbb{A}$ says: "At least one of us is a floop". $\mathbb{B}$ says: "I am a freep".

   (c) $\mathbb{A}$ says "At least one of us is a freep". $\mathbb{B}$ says "I am a freep or both of us are floops".

3. (5 points) Lewis Carroll, the author of *Alice in Wonderland*, was also a logician. He created many logic puzzles. In this problem, we will looking at one of his "syllogism puzzles". In these puzzles your job is to define variables and then write out each of the given premises symbolically as conditional statements. Then write out the contrapositives of each statement. Next, string the symbolic conditional statements together to form the longest chain of syllogisms possible. Finally, you should translate the first hypothesis and final conclusion as a single conditional statement written in ordinary English.

   No one, who is going to a dinner party, ever fails to brush his hair.
   No one looks interesting, if he is untidy.
   Math teachers have no self-control.
   Everyone, who has brushed his hair, looks interesting.
   No one wears a suit, unless he is going to a dinner party.
   A man is always untidy, if he has no self-control.