Math 262
Exam 4 - Practice Problems

1. For each of the following sequences, determine whether the sequence converges or diverges. If a sequence converges, whenever possible, find the value of the limit of the sequence.

   (a) $\left\{\dfrac{n+2}{3n-1}\right\}$

   (b) $\left\{(-1)^n \dfrac{n+2}{3n-1}\right\}$

   (c) $\{ne^{-n}\}$

   (d) $\left\{\dfrac{\cos n}{e^n}\right\}$

   (e) $\{\sqrt[n]{n}\}$

   (f) $\left\{\dfrac{n2^n}{3^n}\right\}$

   (g) $\left\{\left(1+\dfrac{2}{n}\right)^{2n}\right\}$

2. Suppose $a_1 = 1$ and $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{4}{a_n}\right)$

   (a) Compute a_5

   (b) Find $\lim\limits_{n\to\infty} a_n$ [Hint: Let $L = \lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} a_n$. Then $L = \dfrac{1}{2}\left(L + \dfrac{4}{L}\right)$]

3. Determine whether the following series converge or diverge. For those that converge, find the sum of the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2}\left(-\frac{1}{3}\right)^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} 4\left(\frac{1}{2}\right)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4n}{n+2}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{9}{n(n+3)}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4}{3^n}$

4. Use geometric series to express each of the following repeating decimals in fractional form.

   (a) $.11\overline{1}$

   (b) $.7878\overline{78}$

   (c) $.137137\overline{137}$

   (d) $.99\overline{9}$

5. For each of the following series, if the series is positive term, determine whether it is convergent or divergent; if the series contains negative terms, determine whether it is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n(\ln n)^3}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{1+n^{-1}}}{n^2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n - 2}{n^2}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4+2n-1}{n^5+3n^2-20}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}+1}}{n^3}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4}{n+1}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n}{5n+1}\right)^n$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot n}{3^n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4^n}{(2n+1)!}$

   (j) $\displaystyle\sum_{n=1}^{\infty} n^3 e^{-n}$

   (k) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(n!)^2}$

   (m) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[n]{n}}$

6. Estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^4+1}$ to within 0.01

7. Determine the number of terms necessary to estimate the sum of the following series to within 1×10^{-6}

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3}{n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$

8. Find all real values of x for which the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n \cdot 4^n}$ converges.