

1. Find all values for  $p$  for which the integral  $\int_1^{\infty} \frac{1}{x^p} dx$  converges. Note: The parameter  $p$  can be any real number: positive or negative, rational or irrational. Also be careful to check for special cases.

2. (Modified from the 2007 AP Calculus AB exam) Let  $R$  be the region in the first and second quadrants that is bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ . Let  $S$  be the region in the first quadrant bounded above by  $y = \frac{20}{1+x^2}$  and below by the  $x$ -axis. (This problem continues on the following pages.)

(a) Evaluate  $\int \frac{1}{(1+x^2)^2} dx$

(b) Evaluate  $\int_0^3 \frac{1}{(1+x^2)^2} dx$ .

(c) Evaluate  $\int_0^\infty \frac{1}{(1+x^2)^2} dx$ .

(d) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.

(e) Find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.

(f) Find the volume of the solid generated when  $S$  is rotated about the  $x$ -axis.

(g) Find the volume of the solid generated when  $S$  is rotated about the  $y$ -axis.

- (h) Setup, but do not evaluate, an integral expression that gives the length of the boundary curve for the region  $R$ .
- (i) Setup, but do not evaluate, an integral expression that gives the surface area of the solid that is formed when the region  $R$  is rotated about the line  $y = 2$ .
- (j) The region  $R$  is the base of a solid. For this solid, the cross-sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.