1. Fill in each blank with the best word or phrase:

   (a) A sequence $\{a_n\}$ converges to $L$ if _____.

   (b) $\lim_{n\to\infty} r^n = 0$ if _____.

   $\lim_{n\to\infty} |r^n| = \infty$ if _____.

   (c) Let $a \neq 0$. The series $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$

   is called a _____ series.

   it converges and has the sum $S =$ _____ if _____.

   it diverges if _____.

   (d) If $S_n = \dfrac{3n+1}{2n-5}$ then $\sum_{n=1}^{\infty} a_n =$ _____.

   (e) If $S_n = \dfrac{3n+1}{2n-5}$ then $a_5 =$ _____.

   (f) If $a_n = \dfrac{3n+1}{2n-5}$ then $\sum_{n=1}^{\infty} a_n =$ _____.

   (g) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ _____.

   (h) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ _____.

   (i) If $\sum_{n=1}^{\infty} a_n$, is a convergent series and $\sum_{n=1}^{\infty} b_n$ is a divergent series

   then $\sum_{n=1}^{\infty} a_n + b_n$ is a _____ series.

   (j) The series $\sum_{n=1}^{\infty} \dfrac{1}{n^p} = 1 + \dfrac{1}{2^p} + \dfrac{1}{3^p} + \cdots + \dfrac{1}{n^p} + \cdots$ is called a _____ series.

   It converges if _____ and diverges if _____.

2. Determine whether the following sequences converge or diverge. For those that converge, find the limit.

   (a) $\left\{1 + \dfrac{(-1)^n}{n}\right\}$

   (b) $\left\{\dfrac{4^n - 7}{9^n}\right\}$

3. Find the sum of each series:

   (a) $\displaystyle\sum_{n=2}^{\infty} e^{-n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-3)(2n-1)}$

4. Determine whether the following series converge or diverge. Make sure to show all work leading to your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+5}{n^2+7}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

5. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Make sure to show all work leading to your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{3+\ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{n^3+1}$

6. For each of the following power series, find the interval of convergence and the radius of convergence:

(a) $\sum_{n=0}^{\infty} \frac{1}{n3^n}(x+4)^n$

Interval of Convergence: _____

Radius of Convergence: _____

(b) $\sum_{n=0}^{\infty} \frac{1}{n^n}x^n$

Interval of Convergence: _____

Radius of Convergence: _____

7. Find a power series in $x$ that has the given function as its sum. Also find the interval of convergence.

(a) $\frac{1}{1+x^3}$

(b) $\sin\frac{2x}{3}$

8. Find the first four terms of the Taylor series for the function $f(x) = \dfrac{1}{1-x}$ centered at $c = 2$.

9. Find Taylor's formula with remainder for $f(x) = \ln(\cos x)$ with $c = \dfrac{\pi}{6}$ and $n = 3$

10. Approximate the integral $\displaystyle\int_0^1 x \sin(x^3)\, dx$ to six decimal places.