

1. The base of a solid is the region bounded by  $y = x^2$  and  $y = 2 - x^2$ . Find the volume of the solid if every cross section by a plane perpendicular to the  $x$ -axis is:

(a) a square

(b) a semicircle

(c) an equilateral triangle

2. The base of a solid is the region bounded by  $x = \sqrt{5}y^2$ ,  $y = 2$ , and the  $y$ -axis.
- (a) Find the volume of the solid if every cross section by a plane perpendicular to the  $y$ -axis is a semicircle.
- (b) Find the volume of the solid if every cross section by a plane perpendicular to the  $x$ -axis is a semicircle.

**3. Volume of a Wedge**

A plane inclined at an angle of  $45^\circ$  passes through a diameter of the base of a right circular cylinder of radius  $r$ . Find the volume of the region within the cylinder and below the plane. (This volume was first calculated in the third century BCE by Archimedes, who also derived the formula  $V = \frac{4}{3}\pi r^3$  for the volume of a sphere of radius  $r$ . His work on the wedge is found in a manuscript that was discovered in 1906 after having been lost for centuries.)

4. A frustum of a right pyramid is a right pyramid with its top cut off. Let  $V$  be the volume of a frustum of height  $h$  whose base is a square of side  $a$  and top is a square of side  $b$  with  $a > b \geq 0$ . Show that  $V = \frac{h}{3}(a^2 + ab + b^2)$ . (A papyrus dating to the year 1850 BCE indicates that Egyptian mathematicians had discovered this formula almost 4,000 years ago.) [Hint: This problem can be done by modifying the method of example 1 on page 329 of your textbook. It can also be done without using calculus if you begin with the result of this example. Try both ways!]