

Definition: A *linear function* is a function of the form $y = mx + b$. We call m the slope of the line and b the y -intercept.

Not surprisingly, the graph of a linear function is a straight line. Recall that the *graph* of an equation is the set of all points found by evaluating the function at some value from the domain. One particularly nice way to graph a line is to graph the *intercepts* of the line.

Slope: The slope of a line measures how “steep” the line is. It measures how much the line increases or decreases in the y direction when we move one unit the right in the x direction. We can find the slope m of a line by taking any two points on the line $P(x_1, y_1)$ and $Q(x_2, y_2)$ and using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{\text{rise}}{\text{run}}.$$

If the slope $m > 0$, then the line is increasing. If the slope $m < 0$, then the line is decreasing. If the slope $m = 0$, then the line is constant (horizontal).

There are 3 main forms for the equation of a line:

1. General Form: $ax + by = c$
2. Slope/Intercept Form: $y = mx + b$ where m is the slope and b is the y coordinate of the y intercept.
3. Point/Slope Form: $y - y_1 = m(x - x_1)$ where m is the slope and $P(x, y)$ is a point on the line.

If we know two points on a line, we can find the slope of the line using the slope formula. If we know a point on the line and the slope, we can use either the slope/intercept form or the point/slope form equations to find an equation for the line.

Also, we can use algebra to put any linear equation into slope/intercept form so that we can just “read off” the slope and y -intercept of the line.

Horizontal and Vertical Lines:

Two special types of lines that do not quite fit the pattern above are horizontal and vertical lines. Horizontal lines run from left to right, and vertical lines go up and down.

Horizontal lines have equations of the form $y = c$ and so have slope zero (which makes sense since they do not “rise”)

Vertical lines have equations of the form $x = c$ and so have “infinite” slope (which makes sense since they do not “run”)

Parallel and Perpendicular Lines:

Two distinct lines are *parallel* to each other if they never intersect. For this to happen, the lines must have the same slope.

Two lines are *perpendicular* to each other if they form a right angle at the point where they intersect. For this to happen, the slopes of the lines must satisfy the equation:

$$m_2 = -\frac{1}{m_1}, \text{ or, written another way, } m_1 m_2 = -1$$