

**Definition:** The **Logarithm of  $x$  to the base  $b$**  is defined as follows:  $y = \log_b x$  if and only if  $x = b^y$ . for  $x > 0$  and  $b > 0, b \neq 1$ . A logarithm basically asks: “what power would I need to raise the base  $b$  to in order to get  $x$  as the result?”

**Examples:**

Logarithmic Form:	Exponential Form:
(a) $\log_2 8 = 3$	$8 = 2^3$
(b) $\log_2 \frac{1}{2} = -1$	$\frac{1}{2} = 2^{-1}$
(c) $\log_3 81 = 4$	$81 = 3^4$
(d) $\log_8 \frac{1}{64} = -2$	$\frac{1}{64} = 8^{-2}$
(e) $\log_2 -8$ is undefined	$2^y \neq -8$ for any possible $y$ !

**Solving Logarithmic Equations:**

**Note:** Since logarithmic functions are inverses of exponential functions, logarithmic functions are one-to-one. Therefore, as before, we can make use of the definition of a one-to-one function in order to solve basic equations involving logarithmic functions.

**Warning!!** Since  $\log_b x$  is only defined for  $x > 0$ , we will need to check for extraneous solutions. Any value that makes the expression inside a logarithm negative is not a valid solution.

**Examples:**

(a) Suppose  $\log_5 x = 3$ . Find  $x$ .

Since  $\log_5 x = 3$ ,  $x = 5^3 = 125$ .

(b) Suppose  $\log_z 16 = 2$ . Find  $z$ .

Since  $\log_z 16 = 2$ ,  $z^2 = 16$ , so  $z = \pm 4$ . But since we know that  $z > 0$ , then  $z = 4$ .

(c)  $\log_4(3x + 1) = \log_4(2x + 4)$

Since  $\log_4 x$  is one-to-one, we know  $3x + 1 = 2x + 4$

Therefore,  $x = 3$ .

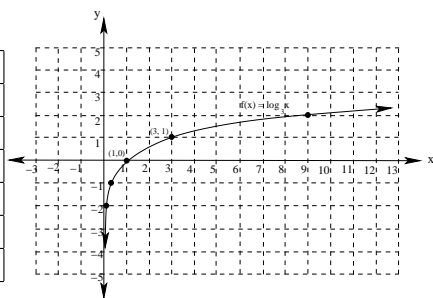
(d)  $\log_7(x^2 + 8x) = \log_7(10x - 8)$

**Notation:** If  $b = 10$ , we abbreviate  $\log_{10} x$  as  $\log x$ . Similarly, if  $b = e$ , we abbreviate  $\log_e x$  as  $\ln x$ .

**Graphs of logarithmic functions:**

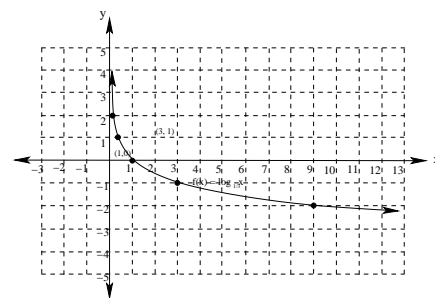
$f(x) = \log_3 x$

$x$	$f(x)$
0	undefined
1	0
3	1
9	2
$\frac{1}{3}$	-1
$\frac{1}{9}$	-2



$f(x) = \log_{\frac{1}{3}} x$

$x$	$f(x)$
0	undefined
1	0
$\frac{1}{3}$	1
$\frac{1}{9}$	2
3	-1
9	-2



**Properties of Logarithmic Graphs:**

1. Domain:  $(0, \infty)$
2. Range:  $(-\infty, \infty)$
3.  $y$ -intercept: none.  $x$  intercept  $(1, 0)$
4. Increasing if  $b > 1$ . Decreasing if  $0 < b < 1$ .

**Examples:** Finding doubling times using logarithms.

- (a) (b)