Math 127
Quadratic Equations Handout

<div align="center">**Quadratic Equations**</div>

Definition: A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$ with $a \neq 0$.

Methods for Solving Quadratic Equations:

1. Factoring [this only works of the quadratic happens to factor]

Example: $3x^2 + 5x - 2 = 0$
- (Factor) $(3x - 1)(x + 2) = 0$ [this only works if one side of the equation is 0!]
- (Split) $3x - 1 = 0$ or $x + 2 = 0$
- (Solve) $3x = 1$, so $x = \frac{1}{3}$ or $x = -2$.

2. Special Form [this works for quadratics of the form $a(x - h)^2 = d$]

Example: $4(x - 5)^2 = 13$
- (divide) [if necessary] $(x - 5)^2 = \frac{13}{4}$
- (square root) $x - 5 = \pm\sqrt{\frac{13}{4}}$
- (add and simplify) [if necessary] $x = 5 \pm \frac{\sqrt{13}}{2}$

3. Completing the Square [this always works]

Example: $2x^2 - 4x - 5 = 0$
- (move constant) $2x^2 - 4x = 5$
- (divide by a if $a \neq 1$) $x^2 - 2x = \frac{5}{2}$
- (add the constant $\left(\frac{b}{2}\right)^2$ to form a perfect square) $\left(\frac{b}{2}\right) = \left(-\frac{2}{2}\right)^2 = 1$
so we have: $x^2 - 2x + 1 = \frac{5}{2} + 1$
- (factor) $(x - 1)^2 = \frac{7}{2}$
- (square root) $x - 1 = \pm\sqrt{\frac{7}{2}}$
- (add and simplify) $x = 1 \pm \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 1 \pm \frac{\sqrt{14}}{2}$

4. The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof: (we will use completing the square to derive this formula)

$ax^2 + bx + c = 0$
$ax^2 + bx = -c$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$
$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac+b^2}{4a^2}}$
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example: $5x^2 - 3x - 1$ [so $a = 5$, $b = -3$, and $c = -1$]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(5)(-1)}}{2(5)} = \frac{3 \pm \sqrt{9 + 20}}{10}$$
$$= \frac{3 \pm \sqrt{29}}{10}$$