Math 262
Extra Credit 3 **Name:**____

Due: Monday, November 9th, 4:30pm

1. This problem will investigate the sequence $\{a_n\}$ that is defined recursively by:
   $a_1 = \sqrt{6} \quad a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$

   (a) Is $a_1 < 3$?

   (b) Show that if we assume that a particular term a_n satisfies the inequality $a_n < 3$, then it follows that $a_{n+1} < 3$.

   Note: (a) and (b) together imply that every term in this sequence is less than 3 [this type of proof is called a "proof by mathematical induction"].

   (c) Since all terms of the sequence $\{a_n\}$ are less than 3, this sequence is said to be ______________.

   (d) Show that $a_{n+1} > a_n$ for all $n \geq 1$. [Hint: Part of your proof should include calculating $(a_{n+1})^2 - (a_n)^2$ and showing that $(a_{n+1})^2 - (a_n)^2 > 0$]

   (e) By (d), $a_1 \leq a_2 \leq \cdots \leq a_n \leq \cdots$. Hence the sequence $\{a_n\}$ is said to be ______________.

   (f) From (c) and (e), we can conclude that the sequence $\{a_n\}$ ______________.

   (g) Let $\lim_{n\to\infty} a_n = A$.

   i. Then $\lim_{n\to\infty} a_{n+1} = A$. Why?

   ii. Use the fact that $\lim_{n\to\infty} a_n = A$ to write the $\lim_{n\to\infty} \sqrt{6 + a_n}$ in terms of A.

   iii. But, $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{6 + a_n}$. Why?

   iv. Use (i), (ii), and (iii) to find A.

   Hence, the work done in this problem proves that the limit of $\{a_n\}$ converges to ______________.