Math 262
Exam 1 - Practice Problems

1. Find the area between the given curves:
   (a) $y = x^2 + 1$ and $y = 3x - 1$
   (b) $y = x^2 - 1$ and $y = 1 - x$ on $[0, 2]$
   (c) $y = x$, $y = 2$, $y + x = 6$, and $y = 0$
   (d) $x = y^2$, $x = 4$
2. A pottery jar has circular cross sections or radius $4 + \sin(\frac{x}{3})$ inches for $0 \leq x \leq \pi$. Sketch the jar and then compute its volume.
3. Let R be the region bounded by $y = x^3$, $y = 0$, and $x = 1$
   (a) Sketch R and then find its area.
   (b) Find the volume of the solid formed by rotating R about the x-axis.
   (c) Find the volume of the solid formed by rotating R about the y-axis.
4. Let R be the region bounded by $y = ax^2$, $y = h$, and the y-axis (where a and h are positive constants). Compute the volume of the solid formed by revolving this region about the y-axis. Show that your answer is equal to half the volume of a cylinder of height h and radius $r = \sqrt{\frac{h}{a}}$. Sketch a picture illustrating the relationship between these two volumes.
5. The great pyramid at Gizeh is 500 feet high rising from a square base with side length 750 feet. Use integration to find the volume of this pyramid.
6. Find the volume of the solid formed by revolving the region bounded by $y = x^2 - 4$ and $y = 4 - x^2$ about the line $x = 2$.
7. Find the volume of the solid formed over the region bounded by $y = x^2 - 4$ and $y = 4 - x^2$ if its cross sections perpendicular to the y-axis are semicircles.
8. Find the volume of the solid formed by revolving the region bounded by $y = x^2$ and the x-axis for $-1 \leq x \leq 1$ about the line $x = -2$.
9. Find the volume of the solid formed by revolving the region bounded by $x = y^2$ and $x = 1$ about the line $y = -2$.
10. Find the volume of the solid formed by revolving the region bounded by $x = y^2$ and $x = 2 + y$ about:
    (a) the line $x = -1$.
    (b) the line $y = -1$.
    (c) the line $x = -2$.
11. Set up the integral for the arc length of each of the following curves on the given interval. You DO NOT need to evaluate the integral.
    (a) $y = x^3 + x$ on $[-2, 5]$
    (b) $y = \tan x$ on $[0, \frac{\pi}{4}]$
    (c) $y = \frac{1}{x^2+1}$ on $[0, 2]$
    (d) $\frac{x^2}{16} + \frac{y^2}{9} = 1$
12. Set up the integral for the surface area of each of the following on the given interval. You DO NOT need to evaluate the integral.
    (a) $y = \sin x$ on $[0, \pi]$ revolved about the x-axis.
    (b) $y = x^3 - 1$ on $[1, 2]$ revolved about the x-axis.
    (c) $y = x^3 - 1$ on $[1, 2]$ revolved about the y-axis.

13. A force of 10 pounds stretches a spring 2 inches. Find the work done in stretching this spring 3 inches beyond its natural length (give your answer in ft-lbs).

14. A cylindrical tank is resting on the ground with its axis vertical; it has a radius of 5 feet and a height of 10 feet. Find the amount of work done in filling this tank with water pumped in from ground level (use $\rho = 62.5/ft^3$ for the density of water).

15. A water tank is in the shape of a right circular cone of altitude 10 feet and base radius 5 feet, with its vertex one the ground (think of an ice cream cone with its point facing down). If the tank is half full, find the work done in pumping all of the water out the *top* of the tank.

16. A chain of density 5 kilograms per meter is hanging down the side of a building that is 100 meters tall. How much work is done if this chain is used to lift a 500 kilogram mass 100 feet up the side of the building?

17. Find the center of mass of the following systems of point masses:

    (a) A 50 kg mass 5 units left of the x-axis, a 30 kg mass 1 unit left of the x-axis and a 20 kg mass 12 units right of the x-axis.

    (b) A 40 kg mass at the point (-2, 5), a 25 kg mass at the point (3, -1) and a 10 lb mass at the point (5, 7).

18. Compute both the mass and the center of mass of a steel rod with density $\rho(x) = 3 - \frac{x}{6}$ for $0 \leq x \leq 6$.

19. Find the centroid of the region R bounded by $y = 4 - x^2$ and the x-axis.

20. Find the centroid of the isosceles triangle formed by the points $(0, 0)$, $(6, 0)$ and $(3, 9)$