Math 262
Practice Problems
Power Series and Taylor Series

1. For each of the following power series, find the interval of convergence and the radius of convergence:

   (a) $\sum_{n=1}^{\infty}(-1)^n n^2 x^n$

   (b) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}(x-3)^n$

   (c) $\sum_{n=1}^{\infty}\frac{n^3}{3^n}(x+1)^n$

   (d) $\sum_{n=1}^{\infty}(-1)^n\frac{10^n}{n!}(x-10)^n$

   (e) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n10^n}(x-2)^n$

2. Use a known series to find a power series in x that has the given function as its sum:

   (a) $x\sin(x^3)$

   (b) $\dfrac{\ln(1+x)}{x}$

   (c) $\dfrac{x-\arctan x}{x^3}$

3. Use a power series to approximate each of the following to within 3 decimal places:

   (a) $\arctan\dfrac{1}{2}$

   (b) $\ln(1.01)$

   (c) $\sin\left(\frac{\pi}{10}\right)$

4. For each of the following functions, find the Taylor Series about the indicated center and also determine the interval of convergence for the series.

   (a) $f(x)=e^{x-1}$, $c=1$

   (b) $f(x)=\cos x$, $c=\dfrac{\pi}{2}$

   (c) $f(x)=\dfrac{1}{x}$, $c=-1$

5. For each of the following functions, find the Taylor Polynomial for the function at the indicated center c. Also find the Remainder term.

   (a) $f(x)=\sqrt{x}$, $c=1$, $n=3$.

   (b) $f(x)=\ln x$, $c=1$, $n=4$.

   (c) $f(x)=\sqrt{1+x^2}$, $c=0$, $n=4$.

6. Estimate each of the following using a Taylor Polynomial of degree 4. Also find the error or your approximation. Finally, find the number of terms needed to guarantee an accuracy or at least 5 decimal places.

   (a) $e^{0.1}$

   (b) $\ln 0.9$

   (c) $\sqrt{1.2}$