Math 262
Quiz 1
Solutions

Name:_____

This is a Take-Home Quiz. You may use your book and course notes, and you may consult with other members of the class, but you may not consult with outside tutors (at least not on these specific problems).

1. (5 points) Find the arc length of the curve given by $y = x^{\frac{2}{3}}$ on $[0, 4]$.

   Recall that the basic formula for finding arc length is: $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

   Here, $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$, so we get:

   $L = \int_0^4 \sqrt{1 + \left[\frac{2}{3}x^{-\frac{1}{3}}\right]^2}\, dx = \int_0^4 \sqrt{1 + \frac{4}{9}x^{-\frac{2}{3}}}\, dx = \int_0^4 \sqrt{1 + \frac{4}{9x^{\frac{2}{3}}}}\, dx$

   We continue to simplify in order to get to a form where we are able to evaluate the integral.

   $= \int_0^4 \sqrt{\frac{9x^{\frac{2}{3}} + 4}{9x^{\frac{2}{3}}}}\, dx = \int_0^4 \frac{\sqrt{9x^{\frac{2}{3}} + 4}}{\sqrt{9x^{\frac{2}{3}}}}\, dx = \int_0^4 \frac{\sqrt{9x^{\frac{2}{3}} + 4}}{3x^{\frac{1}{3}}}\, dx = \int_0^4 \frac{1}{3}x^{-\frac{1}{3}}\sqrt{9x^{\frac{2}{3}} + 4}\, dx$

   We now substitute using $u = 9x^{\frac{2}{3}} + 4$ and $du = 6x^{-\frac{1}{3}}\, dx$

   This gives $\frac{1}{6}\int_4^{9(4^{\frac{2}{3}})+4} \frac{1}{3}u^{\frac{1}{2}}\, du = \frac{1}{18}\frac{3}{2}u^{\frac{3}{2}}\Big|_4^{9(4^{\frac{2}{3}})+4}$

   Evaluating this gives: $\frac{1}{27}\left[\left(9\left(4^{\frac{2}{3}}\right) + 4\right)^{\frac{3}{2}} - 8\right]$

2. (5 points) Find the area of the surface generated by rotating the curve $y = x^3$ between $x = 0$ and $x = 2$ about the x-axis.

   Recall that the basic formula for finding surface area in this situation is: $SA = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx$

   Here, $f'(x) = 3x^2$, so we get:

   $SA = 2\pi \int_0^2 x^3\sqrt{1 + [3x^2]^2}\, dx = 2\pi \int_0^4 x^3\sqrt{1 + 9x^4}\, dx$

   We continue by using the substitution: $u = 1 + 9x^4$ and $du = 36x^3\, dx$

   This gives us the integral: $2\pi \int_1^{145} \frac{1}{36}u^{\frac{1}{2}}\, du = \frac{\pi}{18}\int_1^{145} u^{\frac{1}{2}}\, du = \frac{\pi}{18}\left[\frac{2}{3}u^{\frac{3}{2}}\Big|_1^{145}\right]$

   $= \frac{\pi}{27}\left[145^{\frac{3}{2}} - 1\right]$

   Extra Credit: Derive a formula for the surface area of a sphere of radius r. [Give your work on the back of the quiz or attach additional work]

   Hint: Think of a sphere as being generated by revolving the semicircle given by $f(x) = \sqrt{r^2 - x^2}$ about the x-axis and use a surface integral. The algebra is a little messy, but it works out.