Math 310
Chapter 11 Addendum

Here are a couple of fixes to things that I messed up in class last week:

1. **Claim:** $x\overline{y} + \overline{x}y = (x+y)\overline{(xy)}$

   Proof:

Statement	Reason
$x\overline{y} + \overline{x}y$	Given
$x\overline{x} + x\overline{y} + \overline{x}y + \overline{y}y$	Zero Property
$x\overline{x} + x\overline{y} + y\overline{x} + y\overline{y}$	Commutative Law
$x(\overline{x} + \overline{y}) + y(\overline{x} + \overline{y})$	Distributive Law
$(\overline{x} + \overline{y})x + (\overline{x} + \overline{y})y$	Commutative Law
$(\overline{x} + \overline{y})(x+y)$	Distributive Law
$\overline{(xy)}(x+y)$	De Morgan's Law

2. Finding the **product-of-sums** form for a Boolean expression:

   Recall that given a Boolean Function $F(x_1, x_2, ..., x_n)$, a *minterm* for this function is a product $y_1y_2...y_n$ where each y_i is equal to either x_i or $\overline{x_i}$.

   Similarly, a *maxterm* is a sum of the form $(y_1 + y_2 + ... + y_n)$, where we again assume that each y_i is equal to either x_i or $\overline{x_i}$.

   We will illustrate the process of finding the **product-of-sums** expansion (or the **conjunctive normal form**) by means of an example:

   Example: Consider the Boolean function given by the following table

x	y	z	$F(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

   To find the disjunctive normal for for F, we find the minterm associated with each input that leads to an output of 1. We use x_i when the corresponding input value is a 1, and we use $\overline{x_i}$ when the corresponding input value is 0. Then the three minterms for this function are $\overline{x}y\overline{z}$, $\overline{x}yz$, and $xy\overline{z}$.

   Therefore, $F(x,y,z) = \overline{x}y\overline{z} + \overline{x}yz + xy\overline{z}$ is the disjunctive normal form for F.

   To find the conjunctive normal for for F, we find the maxterm associated with each input that leads to an output of 0 (so precisely the opposite of the rows used before). We use x_i when the corresponding input value is a 0, and we use $\overline{x_i}$ when the corresponding input value is 1. Then the five minterms for this function are $x+y+z$, $x+y+\overline{z}$, $\overline{x}+y+z$, $\overline{x}+y+\overline{z}$, and $\overline{x}+\overline{y}+\overline{z}$.

   Therefore, $F(x,y,z) = (x+y+z)(x+y+\overline{z})(\overline{x}+y+z)(\overline{x}+y+\overline{z})(\overline{x}+\overline{y}+\overline{z})$ is the conjunctive normal form for F.