Math 310
Exam 2 Review Sheet

Section 1.7 - Proof Methods and Strategy

- Understand and be able to utilize Proof by Cases (the Method of Exhaustion).
- Understand how to rule out unnecessary cases (without loss of generality statements).
- Understand and be able to carry out an existence proof (in both constructive and non-constructive forms).
- Understand and be able to carry out a uniqueness proof.
- Understand the role of counterexamples in disproving statements.
- Understand how to form conjectures and know the difference between a conjecture and a theorem.
- Understand how to prove results using "backwards reasoning"

Section 2.1 - Sets

- Understand the definitions of sets, elements, subsets, and proper subsets, as well as symbolic notation for these terms.
- Understand both roster and set builder notation, and be able to determine whether a given set is well defined.
- Understand the definition of set equality and the definition of the cardinality of a set.
- Understand the definitions of the universal set, the empty set, singleton sets, and the power set of a set.
- Understand Venn diagrams and how they can be used to depict the relationships between different sets.
- Understand how to form the Cartesian Product of two sets.

Section 2.2 - Set Operations

- Understand the definition of the set operations: union, intersection, complementation, set difference, and symmetric difference ($\oplus$).
- Understand what it means for two sets to be disjoint, and know how to compute the cardinality of the union of two sets using the principle of inclusion-exclusion.
- Be able to determine which elements are in a set resulting from multiple set operations. Also be able to draw a Venn diagram representing such a set.
- Know the basic set identities (set equalities) and be able to prove them using membership tables, two-column proofs, or paragraph (double containment) proofs.
- Understand how to take the union or the intersection of more than two sets at the same time.
- Understand how to represent sets as binary strings and know how to carry out set operations on binary strings.

Section 4.1 - Mathematical Induction

- Understand the principle of mathematical induction and why it is a valid method of proof.
- Be able to prove the base case of a set of statements of the form $P(n)$ for all $n > 0$
- Be able to prove theorems using mathematical induction.
- Be able to prove statements using string induction.
- Be able to recognize errors in false induction "proofs".

Section 8.1 - Relations and Their Properties

- Know the definition of an binary relation and how it is represented as a set of ordered pairs.
- Understand how to represent relations using tables and arrow diagrams.
- Understand what it means for a relation to be reflexive, irreflexive, symmetric, antisymmetric, or transitive.
- Be able to determine whether a given relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive.
- Be able to combine relations to form new ones using union, intersection, complementation, difference, symmetric difference, and composition.

Section 8.3 Representing Relations

- Understand how to represent a relation using matrices and graphs.
- Given a specific relation, be able to represent it using a matrix and be able to represent is using a directed graph.
- Given a matrix or a directed graph, be able to list the ordered pairs in the relation represented by the matrix (or directed graph).
- Be able to recognize whether a relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive by looking at its matrix.
- Be able to recognize whether a relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive by looking at its directed graph.
- Be able to apply the operations union, intersection, complementation, difference, symmetric difference, and composition to matrices of relations and to directed graphs of relations.