

**Math 310****Examples of 2-Column Proofs**

1. Recall the example that we started in class on Friday:

$$\begin{array}{l} (\neg r \vee \neg f) \rightarrow (h \wedge d) \\ h \rightarrow t \\ \neg t \\ \hline \therefore r \end{array}$$

**Proof:**

Statement	Reason
1. $h \rightarrow t$	Premise
2. $\neg t$	Premise
3. $\neg h$	Modus Tollens using (1) and (2)
4. $\neg h \vee \neg d$	Addition and (3)
5. $\neg(h \wedge d)$	De Morgan's Law and (4)
6. $(\neg r \vee \neg f) \rightarrow (h \wedge d)$	Premise
7. $\neg(\neg r \vee \neg f)$	Modus Tollens using (5) and (6)
8. $r \wedge f$	De Morgan's Law and Double Negation and (7)
9. $r$	Simplification and (8)

2. Use a 2-Column Proof to verify the following argument:

$$\begin{array}{l} p \rightarrow q \\ \neg(q \wedge r) \\ r \\ \hline \therefore \neg p \end{array}$$

**Solution:**

Statement	Reason
1. $\neg(q \wedge r)$	Premise
2. $\neg q \vee \neg r$	DeMorgan's Law applied to (1)
3. $r$	Premise
4. $\neg(\neg r)$	Double Negation and (3)
5. $\neg q$	Disjunctive Syllogism applied to (2) and (4)
6. $p \rightarrow q$	Premise
7. $\neg q \rightarrow \neg p$	Contraposition and (6)
8. $\neg p$	Modus Ponens applied to (5) and (7)

3. Write a 2-column proof to verify the following argument:

$$\begin{array}{l} t \rightarrow p \\ s \vee t \\ p \rightarrow q \\ \neg q \\ \hline \therefore s \end{array}$$

**Solution:**

Statement	Reason
1. $t \rightarrow p$	Premise
2. $p \rightarrow q$	Premise
3. $t \rightarrow q$	Hypothetical Syllogism applied to (1) and (2)
4. $\neg q$	Premise
5. $\neg t$	Modus Tollens applied to (3) and (4)
6. $s \vee t$	Premise
7. $s$	Disjunctive Syllogism applied to (5) and (6)