Math 310
Project 1 Handout
Due: 9/30/2009

Instructions: This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with at most one other student on this assignment. If you decide to work with another student, you may turn in a combined paper with both your names listed.

1. (10 points) In *Misère NIM*, the game begins with a pile of n stones. On their turn, a player can take either 1, 2, or 3 stones. However, unlike the version presented in class, in this case, the player that takes the last stone **loses** the game.

   (a) Which player has a winning strategy if the game starts with 7 stones?

   (b) Do a complete analysis of *Misère NIM* as the game is defined above. That is, determine which player has a winning strategy for each possible value of n [Hint: split into cases].

   (c) Do a complete analysis of *Misère NIM* if a player can take either 1, 2, 3, or 4 stones on their turn.

2. (10 points) This part of the project involves a famous set-theoretic problem called Russell's Paradox. According to Russel's definitions, a set A is called **normal** if A is *not* an element of itself. Similarly, a set is **abnormal** if it *is* an element of itself.

   (a) Give an example of a set that is normal.

   (b) Give an example of a set that is abnormal.

   (c) Let $\mathcal{N} = \{A \,|\, A \text{ is a set that is normal }\}$ and let $\mathcal{A} = \{A \,|\, A \text{ is a set that is abnormal }\}$. Is $\mathcal{N}$ a normal set or an abnormal set? How about $\mathcal{A}$? Explain how this leads to a paradox and comment on what caused things to go wrong.

3. (10 points)

   (a) Suppose that a fast food restaurant sells chicken nuggets in packs of 4, 7, or 9. What is the largest number of chicken nuggets that you **cannot** buy *exactly* (Justify your answer).

   (b) Now suppose that a different restaurant sells chicken nuggets in packs of 4 or 15. What is the largest number of chicken nuggets that you **cannot** buy *exactly* (Justify your answer).

   (c) Finally suppose that a different restaurant sells chicken nuggets in packs of 6, or 9. Is there a largest number of chicken nuggets that you **cannot** buy *exactly*? (Justify your answer).

   Extra Credit: Given a restaurant that sells chicken nuggets in packs of size $n_1, n_2, ..., n_k$, what needs to be true about $n_1, n_2, ..., n_k$ in order for there to be a largest number of chicken nuggets that you cannot buy exactly?