Math 127
Exam 1 Practice Problems

1. Express the following number in scientific notation: .0001093

   1.093×10^{-4}

2. Express the following number in ordinary decimal notation: 4.03267×10^4

   40326.7

3. True or False:

   (a) $(a+b)c = ac + bc$
   Solution: True (This is one of the forms of the distributive property).

   (b) If $ab = 1$, then either $a = 1$ or $b = 1$ or both a and b equal 1
   Solution: False. For example, if $a = \frac{1}{2}$ and $b = 2$, then $ab = 1$.

   (c) $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a+c}{b+d}$
   Solution: False. For example, if $a = c = 3$, and $b = d = 1$, then $\frac{a}{b} + \frac{c}{d} = \frac{3}{1} + \frac{3}{1} = 6$, while $\frac{a+c}{b+d} = \frac{3+3}{1+1} = \frac{6}{2} = 3$.

   (d) $\dfrac{a+c}{b} = \dfrac{a}{b} + \dfrac{c}{b}$
   Solution: True

   (e) $5^{\frac{1}{2}} = \dfrac{1}{5^2}$
   Solution: False. $5^{\frac{1}{2}} = \sqrt{5}$, while $\frac{1}{5^2} = \frac{1}{25}$.

   (f) $(a+b)^2 = a^2 + b^2$
   Solution: False. $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$.

   (g) $x = 0$ is a solution to the equation $\dfrac{x^2}{x} = 0$
   Solution: False. Division by zero is not defined for real numbers.

4. Simplify the following:

   (a) $\left(\dfrac{3}{4}\right)^{-2}$
   Solution:
   $$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}$$

   (b) $8^{\frac{4}{3}}$
   Solution:
   $$8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$$

   (c) $\left(\dfrac{y^{12}}{25z^4}\right)^{-\frac{3}{2}}$
   $$\left(\frac{y^{12}}{25z^4}\right)^{-\frac{3}{2}} = \left(\frac{25z^4}{y^{12}}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25z^4}{y^{12}}}\right)^3 = \left(\frac{5z^2}{y^6}\right)^3 = \frac{125z^6}{y^{18}}$$

(d) $\sqrt[5]{32x^{11}y^{14}z^8}$
Solution:
$\sqrt[5]{32x^{11}y^{14}z^8} = \sqrt[5]{32x^{10}xy^{10}y^4z^5z^3} = 2x^2y^2z\sqrt[5]{xy^4z^3}$

(e) $\left(\dfrac{(5xyz)^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1}$
Solution:
$\left(\dfrac{(5xyz)^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1} = \left(\dfrac{25x^2y^2z^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1} = \left(\dfrac{25x^{2+2}y^{2-2}z^{2-2+4}}{2}\right)^{-1}\left(\dfrac{25x^4z^4}{2}\right)^{-1} = \dfrac{2}{25x^4z^4}$

5. Rationalize the denominator in the following expressions:

(a) $\dfrac{3x}{\sqrt[3]{x}}$
Solution:
$\dfrac{3x}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{3x\sqrt[3]{x^2}}{x} = 3\sqrt[3]{x^2}$

(b) $\dfrac{2x+3}{\sqrt{2x}-1}$
Solution:
$\dfrac{2x+3}{\sqrt{2x}-1} \cdot \dfrac{\sqrt{2x}+1}{\sqrt{2x}+1} = \dfrac{(2x+3)\sqrt{2x}+1}{(2x-1)}$

6. Perform the indicated operations and simplify:

(a) $3(2x^3 - x^2 + 5x) - 2x(3x^3 - 2x^2 + 5x - 3)$
Solution:
$3(2x^3-x^2+5x)-2x(3x^3-2x^2+5x-3) = 6x^3-3x^2+15x-6x^4+4x^3-10x^2+6x = -6x^4+10x^3-13x^2+21x$

(b) $(2x^2 + 3x - 2)(x - 2)$
Solution:
$(2x^2 + 3x - 2)(x - 2) = 2x^3 + 3x^2 - 2x - 4x^2 - 6x + 4 = 2x^3 - x^2 - 8x + 4$

(c) $(2x + 1)^3$
Solution:
$(2x + 1)^3 = (2x + 1)(2x + 1)(2x + 1) = (4x^2 + 4x + 1)(2x + 1) = 8x^3 + 4x^2 + 8x^2 + 4x + 2x + 1 = 8x^3 + 12x^2 + 6x + 1$, or, use the expansion formula for cubes, which yields the same result.

(d) $(x^{\frac{1}{3}} + y^{\frac{1}{3}})(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$
Solution:
$= x + x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} + y$
$= x + y$

7. Factor each of the following expressions completely:

(a) $2x^2 + x - 6$
Solution: $(2x - 3)(x + 2)$

(b) $50x^2 + 45x - 18$
Solution:
$(50 \cdot -18 = -900 = -1 \cdot 10 \cdot 10 \cdot 9 = -1 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 60 \cdot -15)$
$= 50x^2 + 60x - 15x - 18 = 10x(5x + 6) - 3(5x + 6) = (5x + 6)(10x - 3)$

(c) $9x^2 - 49y^6$
Solution: $(3x - 7y^3)(3x + 7y^3)$

(d) $8x^3 - y^3$
Solution: $(2x - y)(4x^2 + 2xy + y^2)$

(e) $6x^3y - 27x^2y - 15xy$
Solution: $(3xy)(2x^2 - 9x - 5) = (3xy)(2x + 1)(x - 5)$

(f) $3x^3 + x^2 - 3x - 1$
Solution: $x^2(3x + 1) - 1(3x + 1) = (x^2 - 1)(3x + 1) = (x + 1)(x - 1)(3x + 1)$

(g) $x^6 - 1$
Solution:
$x^6 - 1 = (x^2)^3 - (1)^3$, so by the difference of cubes factoring formula:
$= (x^2 - 1)(x^4 + x^2 + 1)$
$= (x + 1)(x - 1)(x^4 + x^2 + 1)$

8. Simplify the following expressions:

(a) $\dfrac{3x^2 - 10x + 3}{x^2 - 1} \cdot \dfrac{x^2 + x - 2}{x^2 - 9}$
Solution:
$$\frac{3x^2 - 10x + 3}{x^2 - 1} \cdot \frac{x^2 + x - 2}{x^2 - 9} = \frac{(3x - 1)(x - 3)}{(x + 1)(x - 1)} \cdot \frac{(x + 2)(x - 1)}{(x + 3)(x - 3)} = \frac{(3x - 1)(x + 2)}{(x + 1)(x + 3)}$$

(b) $\dfrac{2x^2 + 4}{2x^2 + 7x - 4} - \dfrac{x - 1}{x + 4}$
Solution:
$$\frac{2x^2 + 4}{2x^2 + 7x - 4} - \frac{x - 1}{x + 4} = \frac{2x^2 + 4}{(2x - 1)(x + 4)} - \frac{x - 1}{x + 4} = \frac{2x^2 + 4}{(2x - 1)(x + 4)} - \frac{(2x - 1)(x - 1)}{(2x - 1)(x + 4)}$$
$$= \frac{2x^2 + 4 - (2x^2 - 3x + 1)}{(2x - 1)(x + 4)} = \frac{2x^2 + 4 - 2x^2 + 3x - 1}{(2x - 1)(x + 4)} = \frac{3x + 3}{(2x - 1)(x + 4)} = \frac{3(x + 1)}{(2x - 1)(x + 4)}$$

(c) $\dfrac{\frac{1}{x} + \frac{3}{x-2}}{\frac{4}{x-1} - \frac{2}{x-2}}$
Solution:
$$\frac{\frac{1}{x} + \frac{3}{x-2}}{\frac{4}{x-1} - \frac{2}{x-2}} \cdot \frac{x(x - 1)(x - 2)}{x(x - 1)(x - 2)}$$
$$= \frac{(x - 1)(x - 2) + 3(x)(x - 1)}{4(x)(x - 2) - 2(x)(x - 1)} = \frac{(x - 1)(x - 2 + 3x)}{x[4(x - 2) - 2(x - 1)]}$$
$$= \frac{(x - 1)(4x - 2)}{x[4x - 8 - 2x + 2]} = \frac{2(x - 1)(2x - 1)}{x(2x - 6)} = \frac{2(x - 1)(2x - 1)}{2x(x - 3)} = \frac{(x - 1)(2x - 1)}{x(x - 3)}$$

(d) $\dfrac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h}$
Solution:

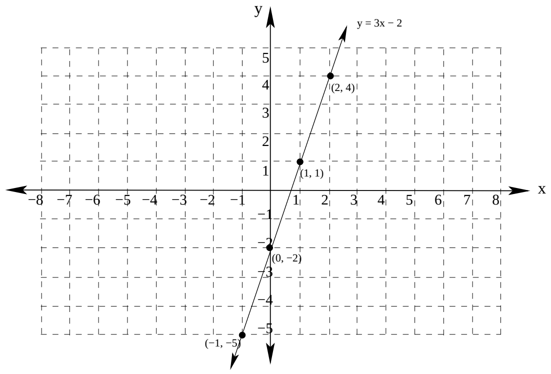
(e) $$\frac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h} = \frac{\frac{3(2x+1)}{(2x+2h+1)(2x+1)} - \frac{3(2x+2h+1)}{(2x+1)(2x+2h+1)}}{h}$$

(f) $$= \frac{(6x + 3) - (6x + 6h + 3)}{(2x + 2h + 1)(2x + 1)} \cdot \frac{1}{h} = \frac{-6h}{(2x + 1)(2x + 2h + 1)(h)}$$

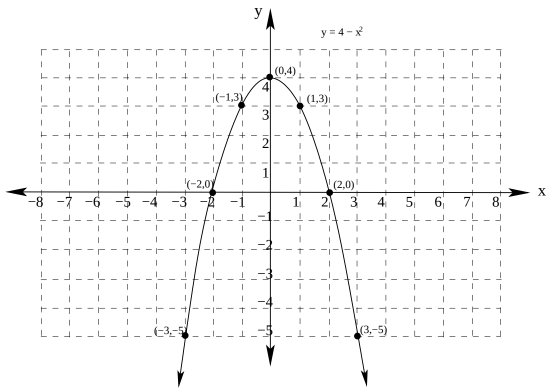
(g) $$= \frac{-6}{(2x + 1)(2x + 2h + 1)}$$

9. Sketch the graphs of the following equations:

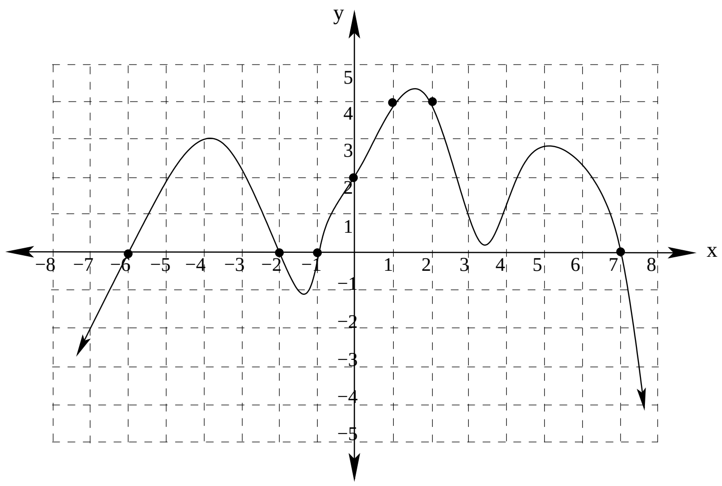
(a) $y = 3x - 2$



(b) $y = 4 - x^2$



10. Based on the graph given below:



(a) Find the coordinates of all x intercepts.
From the graph, we see that the x-intercepts are at the points: $(-6, 0), (-2, 0), (-1, 0)$ and $(7, 0)$

(b) Find the coordinates of all y intercepts.
From the graph, we see that the y-intercept is at the point: $(0, 2)$

(c) Find the x-value(s) when $y = 4$
From the graph, we see that the x-values corresponding to $y = 4$ are $x = 1$ and $x = 2$.