Math 323 Final Exam Practice Problem Solutions

1. Given the vectors $\vec{a} = \langle 1, 2, 3\rangle$ and $\vec{b} = \langle -1, 1, 2\rangle$, compute the following:

   (a) $3\vec{a} - 2\vec{b}$
   Solution:
   $3\vec{a} - 2\vec{b} = \langle 3, 6, 9\rangle + \langle 2, -2, -4\rangle = \langle 5, 4, 5\rangle$.

   (b) $\vec{a} \times \vec{b}$.
   Solution:
   $$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{vmatrix} = \vec{i}(4-3) - \vec{j}(2+3) + \vec{k}(1+2) = \langle 1, -5, 3\rangle.$$

   (c) A unit vector in the direction opposite $\vec{a}$.
   Solution:
   $\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{\langle -1,-2,-3\rangle}{\sqrt{1+4+9}} = \langle \frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}\rangle$.

   (d) The component of $\vec{a}$ along $\vec{b}$.
   Solution:
   $comp_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|} = \frac{\langle 1,2,3\rangle\cdot\langle -1,1,2\rangle}{\sqrt{1+1+4}} = \frac{(-1+2+6)}{\sqrt{6}} = \frac{7}{\sqrt{6}}$.

   (e) The projection of $\vec{b}$ along $\vec{a}$.
   Solution:
   $proj_{\vec{a}}\vec{b} = \frac{\vec{b}\cdot\vec{a}}{\|\vec{a}\|^2}\vec{b} = \frac{7}{\sqrt{14}^2}\langle -1, 1, 2\rangle = \langle -\frac{1}{2}, \frac{1}{2}, 1\rangle$.

   (f) The angle between $\vec{a}$ and $\vec{b}$
   Solution:
   $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{7}{\sqrt{14}\sqrt{6}} = \frac{7}{2\sqrt{21}}$, so $\theta = \cos^{-1}(\frac{7}{2\sqrt{21}}) \approx 40.6°$.

   (g) A vector which is perpendiclar to $\vec{b}$
   Solution:
   There are many possible answers here.
   One possibility is $\vec{v} = \langle 1, 1, 0\rangle$, since then $\vec{b}\cdot\vec{b} = \langle 1, 1, 0\rangle \cdot \langle -1, 1, 2\rangle = -1 + 1 + 0 = 0$.

2. Evaluate the following limit or show it doesn't exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(2x^2+2y^2)}{x^2+y^2}$

   Solution:
   Assume that $(x, y) \to (0, 0)$ along some curve given by $x = f(y)$.
   Then $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(2x^2+2y^2)}{x^2+y^2} = \lim_{(f(y),y)\to(0,0)} \frac{\sin(2(f(y))^2+2y^2)}{(f(y))^2+y^2}$. Using L'Hopital's Rule:
   $$\lim_{(x,y)\to(0,0)} \frac{\sin(2x^2+2y^2)}{x^2+y^2} = \lim_{y\to 0} \frac{(4f(y)f'(y)+4y)\cos(2(f(y))^2+2y^2)}{2f(y)f'(y)+2y} = \lim_{y\to 0} 2\cos(2(f(y))^2+2y^2) = 2\cos(0) = 2.$$
   If $(x, y) \to (0, 0)$ along some curve given by $y = g(x)$, a similar computation shows that the limit is 2 in this case as well. Therefore the limit along any path is 2, so the limit exists and is equal to 2.

3. A projectile is fired with initial speed $v_0 = 80$ feet per second from a height of 6 feet, and at an angle of $\frac{\pi}{4}$ above the horizontal. Assuming that the only force acting on the obeject is gravity, find its maximum altitude, horizontal range, and speed at impact.

   Solution:
   We are given that $v_0 = 80\frac{ft.}{sed.}$, $h = 6ft.$, $\theta = \frac{\pi}{4}$, and $g = -32\frac{ft.}{sec.^2}$. Then $\vec{a}(t) = \langle 0, -32\rangle$, $\vec{v}(t) = \langle v_0\cos\theta, v_0\sin\theta - 32t\rangle = \langle 40\sqrt{2}, 40\sqrt{2} - 32t\rangle$, and $\vec{r}(t) = \langle (v_0\cos\theta)t, (v_0\sin\theta)t - 16t^2 + h\rangle = \langle 40\sqrt{2}t, 40\sqrt{2}t - 16t^2 + 6\rangle$.

   The maximum altitude occurs when $40\sqrt{2} - 32t = 0$, or when $t = \frac{40\sqrt{2}}{32}$. Plugging this tim into the vertical coordinate function of $\vec{r}(t)$ gives the maximum altitude: $40\sqrt{2}(\frac{40\sqrt{2}}{32}) - 16(\frac{40\sqrt{2}}{32})^2 + 6 = 56$ feet.

   The horizontal range is giving by finding the impact time, when $40\sqrt{2}t - 16t^2 + 6 = 0$. The positive solution of this quadratic function is $t \approx 3.6386$, so the horizontal range is $40\sqrt{2}(3.6386) \approx 205.83$ feet.

   The speed at impact is the magnitude of the velocity function $\vec{v}(t)$ at time $t \approx 3.6386$. $\|\vec{v}(3.6386)\| = \|\langle 40sqrt2, 40\sqrt{2} - 32(3.6386)\rangle\| = 82.365\frac{ft.}{sec.}$.

4. Let $f(x,y)=\sqrt{x^2+y^2}$. Find f_{xx} and f_{yx}.

Solution:

$f_x = \frac{1}{2}(x^2+y^2)^{\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2}(x^2+y^2)^{\frac{1}{2}}(2y) = \frac{y}{\sqrt{x^2+y^2}}$

$f_{xx} = (x^2+y^2)^{\frac{-1}{2}} + (x)(\frac{-1}{2})(x^2+y^2)^{\frac{-3}{2}}(2x)$

$f_{yx} = (y)(\frac{-1}{2})(x^2+y^2)^{\frac{-3}{2}}(2x)$

5. (a) Find the equation of the tangent plane and normal line to the surface $z=\sqrt{x^2+y^2}$ at the point (3,4,5).

Solution:

Notice that since $z=f(x,y)=\sqrt{x^2+y^2}$, $f_x = \frac{1}{2}(x^2+y^2)^{\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$, and $f_y = \frac{1}{2}(x^2+y^2)^{\frac{1}{2}}(2y) = \frac{y}{\sqrt{x^2+y^2}}$. Then the tangent plane to $z=f(x,y)$ at the point $(3,4,5)$ is given by $z=f(3,4)+f_x(3,4)(x-3)+f_y(3,4)(y-4) = 5+\frac{3}{5}(x-3)+\frac{4}{5}(y-4)$. The normal line to this plane can be found by parameterising the line through $(3,4,5)$ in the direction of the normal vector to the tangent plane, which is given by $\vec{n}=\langle f_x(3,4), f_y(3,4), -1\rangle = \langle \frac{3}{5}, \frac{4}{5}, -1\rangle$.

Thus the normal line is given by $\ell : \begin{cases} x(t) = 3+\frac{3}{5}t \\ y(t) = 4+\frac{4}{5}t \\ z(t) = 5-t \end{cases}$

(b) Use the plane you found in (a) to estimate the value of z when $x=4$ and $y=4$. How good is the approximation?

Solution:

Approximating using the tangent plane formula derived in part (a) above, we get $f(4,4)\approx 5+\frac{3}{5}(4-3)+\frac{4}{5}(4-4) = 5+\frac{3}{5} = 5.6$

The actual function value is: $f(4,4)=\sqrt{4^4+4^2}=\sqrt{32}\approx 5.65685$. The approximation appears to be good to within about 1 decimal place.

(c) Find the direction and magnitude of the maximum rate of change of $z=f(x,y)=\sqrt{x^2+y^2}$ at (3,4,5).

Solution:

The maximum rate of change in in the direction of the gradient at $(3,4,5)$, namely, $\langle \frac{3}{5}, \frac{4}{5}\rangle$. The maxnitude of the maximum rate of change is $\|\langle \frac{3}{5}, \frac{4}{5}\rangle\| = \sqrt{(\frac{3}{5})^2+(\frac{4}{5})^2} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$.

6. Let $T(x,y)=3x^2y+xe^y$ denote the temperature of a metal plate at the point (x,y). A thermometer is placed at the point $P=(1,0)$. At what rate is the temperature changing as the thermometer is moved from P towards the point (2,-3)?

Solution:

The vector that gives the direction of movement from P to Q is $\vec{v}=\langle 2-1, -3-0\rangle = \langle 1,-3\rangle$. A unit vecot in this direction is: $\vec{u}=\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\rangle$. $\nabla T = \langle 6xy+e^y, 3x2+xe^y\rangle$, so $\nabla T(1,0) = \langle 0+e^0, 3(1)^2+1e^0\rangle = \langle 1,4\rangle$. Therefore, $D_{\vec{u}}f(1,0) = \nabla T(1,0)\cdot\vec{u} = \langle 1,4\rangle\cdot\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\rangle = \frac{1}{\sqrt{10}} - \frac{12}{\sqrt{10}} = \frac{-11}{\sqrt{10}}$.

7. Use the Chain Rule to find:

(a) $g'(t)$ where $g(t)=f(x(t),y(t))$, $f(x,y)=x^2y+y^2$, $x(t)=e^{4t}$, and $y(t)=\sin t$.

Solution:

By the Chain Rule, $g'(t) = f_x(x(t),y(t))(x'(t)) + f_y(x(t),y(t))(y'(t)) = (2xy)(4e^{4t}) + (x^2+2y)(\cos t) = (2(e^{4t})(\sin t))(4e^{4t}) + ((e^{4t})^2+2\sin t)(\cos t) = 8e^{8t}\sin t + e^{8t}\cos t + 2\sin t\cos t$.

(b) g_u and g_v where $g(u,v)=f(x(u,v),y(u,v))$, $f(x,y)=4x^2-y$, $x(u,v)=u^3v+\sin u$, and $y(u,v)=4v^2$.

Solution:

By the Chain Rule, $g_u = (f_x)(x_u)+(f_y)(y_u) = (8x)(3u^2v+\cos u)+(-1)(0) = 8(u^3v+\sin u)(3u^2v+\cos u)$ and $g_v = (f_x)(x_v)+(f_y)(y_v) = (8x)(u^3)+(-1)(8v) = 8(u^3v+\sin u)(u^3)-8v$.

8. Use implicit differentiation to find $\frac{dz}{dx}$ if $x^2z-y^2x+3y-z=-4$.

Solution:

Recall that $\frac{dz}{dx} = \frac{-F_x}{F_z}$. Here, $F_x = 2xz-y^2$, and $F_z = x^2-1$. Therefore, $\frac{dy}{dz} = \frac{y^2-2xz}{x^2-1}$.

9. Let $f(x, y) = -\frac{1}{3}x^3 + xy - 12y + \frac{1}{2}y^2$. Find and classify all critical points of $f(x, y)$.

Solution:

To find and classify critical points of a functon of two variables, we first find all points where either both partials are zero, or where one of the partials is undefined. Notice that $f_x = -x^2 + y$ and $f_y = x - 12 + y$, which are defined everywhere, so critical points occur when $-x^2 + y = 0$, that is, when $y - x^2$, and when $x - 12 + y = 0$. That is, when $y = 12 - x$. Combining these, we have $x^2 = 12 - x$, or $x^2 + x - 12 = (x + 4)(x - 3) = 0$. Therefore, the critical points occur when $x = -4$ or $x = 3$, which, since $y = 12 - x$, gives two critical points: $(-4, 16)$, and $(3, 9)$.

We classify these critical points using the second derivative test. Since $f_{xx} = -2x$, $f_{yy} = 1$, and $f_{xy} = f_{yx} = 1$, then $D(x, y) = f_{xx}f_{yy} - (f_{xy}f_{yx})^2 = -2x - 1$. Then $D(-4, 16) = -2(-4) - 1 = 7$, and since $7 > 0$ and $f_{xx} > 0$, we know that $(-4, 16)$ is a local min. Also, $D(3, 9) = -2(3) - 1 = -7$, and $-7 < 0$, so $(3, 9)$ is a saddle point.

10. Find the maximum value of $f(x, y, z) = x + 2y - 4z$ on the sphere $x^2 + y^2 + z^2 = 21$.

Solution:

Since we want to maximize one function with respect to a given constraint equation, this problem can be solved using LaGrange multipliers.

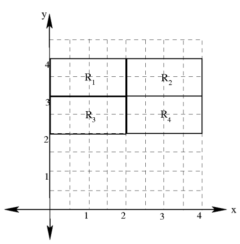
Note that $\nabla f = \langle 1, 2, -4\rangle$ and $\nabla g = \langle 2x, 2y, 2z\rangle$. Then we set $\langle 1, 2, -4\rangle = \lambda\langle 2x, 2y, 2z\rangle$ and solve, yielding $2\lambda x = 1$, $2\lambda y = 2$, and $2\lambda z = -4$. Thus $x = \frac{1}{2\lambda}$, $y = \frac{1}{\lambda}$, and $z = \frac{-2}{\lambda}$. Plugging these into our constraint equation gives $(\frac{1}{2\lambda})^2 + (\frac{1}{\lambda})^2 + (\frac{-2}{\lambda})^2 = 21$, or $\frac{1}{4\lambda^2} + \frac{1}{\lambda^2} + \frac{4}{\lambda^2} = 21$. Therefore, $\frac{1+4+16}{4\lambda^2} = 21$, so $21 = (21)(4\lambda^2)$. Hence $4\lambda^2 = 1$, or $\lambda^2 = \frac{1}{4}$. Thus $\lambda = \pm\frac{1}{2}$.

If $\lambda = \frac{1}{2}$, then $x = 1, y = 2, z = -4$, and $f(1, 2, -4) = 1 + 4 + 16 = 21$. If $\lambda = \frac{-1}{2}$, then $x = -1, y = -2, z = 4$, and $f(-1, -2, 4) = -1 - 4 - 16 = -21$. Therefore the maximum value of 21 occurs at the point $(1, 2, -4)$.

11. Compute a Riemann sum to estimate the volume of the function $f(x, y) = 3x^2 + 4y$ on the region $0 \le x \le 4$, $2 \le y \le 4$ partitioned into $n = 4$ equal sized rectangles, and evaluating each rectangle at its midpoint.

Solution:

There are actually a few reasonable ways to subdivide R into four equal sized rectangles. We will use the following:



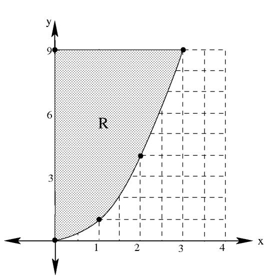
Notice that $V \approx \displaystyle\sum_{i=1}^{4} f(M_i)\Delta A_i$, where $M_1 = (1, \frac{7}{2})$, $M_2 = (3, \frac{7}{2})$, $M_3 = (1, \frac{5}{2})$, $M_4 = (3, \frac{5}{2})$, and $\Delta A_i = 2$ for every i.

Then $V \approx (3(1)^2 + 4\left(\frac{7}{2}\right))2 + (3(3)^2 + 4\left(\frac{7}{2}\right))2 + (3(1)^2 + 4\left(\frac{5}{2}\right))2 + (3(3)^2 + 4\left(\frac{5}{2}\right))2 = (17)2 + (41)2 + (13)2 + (27)2 = 196$

12. Reverse the order of integration in the following iterated integral: $\displaystyle\int_0^9 \int_0^{\sqrt{y}} f(x, y)\, dx\, dy$.

Solution:

Notice that we have $0 \le y \le 9$ and $0 \le x \le \sqrt{y}$. (If $x = \sqrt{y}$, then $y = x^2$)



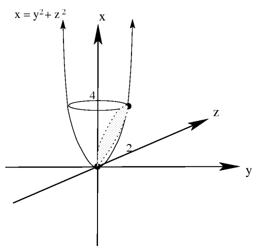
Then, reversing the order of integration, we have: $\displaystyle\int_0^3 \int_{x^2}^{9} f(x, y)\, dy\, dx$

13. Find an iterated triple integral which gives the volume of the solid bounded by the graphs of $x = y^2 + z^2$ and $x = 2z$. DO NOT EVALUATE THE INTEGRAL.

Solution:

Notice that $x = y^2 + z^2$ is a paraboloid that opens along the positive x-axis, and $x = 2z$ is a plane. To understand the region this volume sits over, we must find the intersection of these two surfaces:

If $y^2 + z^2 = 2z$, then $y^2 + z^2 - 2z = 0$, so $y^2 + z^2 - 2z + 1 = 1$, or $y^2 + (z-1)^2 = 1$. Therefore, these two surfaces intersect in an ellipse that sits over the circle of radius 1 centered at the point $(0, 1)$ in the yz-plane.



Since the volume we want to compute sits above the paraboloid $x = y^2 + z^2$, below the plane $x = 2z$, and inside the circle $y^2 + (z-1)^2 = 1$ in the yz-plane, we have the following:

$y^2 + z^2 \le x \le 2z$, $-\sqrt{2z - z^2} \le y \le \sqrt{2z - z^2}$, and $0 \le z \le 2$

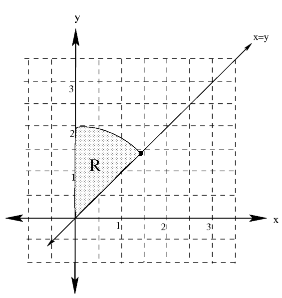
Hence the intgral is given by $V = \int_0^2 \int_{-\sqrt{2z-z^2}}^{\sqrt{2z-z^2}} \int_{y^2+z^2}^{2z} 1 \, dx \, dy \, dz$

14. Convert the following integral into an iterated integral in spherical coordinates.

DO NOT EVALUATE THE INTEGRAL: $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z \, dz \, dy \, dx.$

Solution:

From the given limits of integration, we have: $0 \le x \le \sqrt{2}$, $x \le y \le \sqrt{4 - x^2}$, and $0 \le z \le \sqrt{4 - x^2 - y^2}$. Looking at the outermost two variables, we have an integral over the following region:



If $z = \sqrt{4 - x^2 - y^2}$, then $z^2 = 4 - x^2 - y^2$, or $x^2 + y^2 + z^2 = 4$, so the solid we are integrating over is bounded below by the xy-plane and above by the sphere of radius 2 centered at the origin.

Finally, the integrand $z = \rho \cos \phi$, and the differential is given by $dV = \rho^2 \sin \phi$.

Thus, the following integral represents the original integral translated into spherical coordinates.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\pi} \int_0^2 \rho \cos \phi \left(\rho^2 \sin \phi\right) \, d\rho \, d\phi \, d\theta$$

15. Let $\vec{F}(x, y, z) = \langle 2xy, \, x^2 - 2z, \, 12z - 2y \rangle$.

(a) Show that $\vec{F}$ is conservative by finding a potential function for $\vec{F}$.

Solution:

We begin by antidifferentiating each component of the vector field:

$f(x,y,z) = x^2y + g(y,z)$
$f(x,y,z) = x^2y - 2yz + h(x,z)$
$f(x,y,z) = 6z^2 - 2yz + k(x,y)$

From this, we see that $f(x,y,z) = x^2y - 2yz + 6z^2$ is a potential function for this vector field.

(b) Evaluate $\int_{(0,0,0)}^{(1,1,2)} \vec{F} \cdot d\vec{r}$.

Solution:

Using the Fundamental Theorem of line integrals:
$\int_{(0,0,0)}^{(1,1,2)} \vec{F} \cdot d\vec{r} = f(1,1,2) - f(0,0,0) = [(1 - 4 + 24) - (0)] = 21$

16. Set up an iterated integral for $\iint_S g(x,y,z)\, dS$ where $g(x,y,z) = x^2z$ and S is the upper half of the ellipsoid $x^2 + 4y^2 + z^2 = 4$. DO NOT EVALUATE THE INTEGRAL.
Solution:

Recall that $\iint_S g(x,y,z)\, dS = \iint_R g(x,y,z) \sqrt{f_x^2 + f_y^2 + 1}\, dA$, where R is the region in the plane under the surface S, and the surface S is given by $z = f(x,y)$.
In this case, the region R in the plane can be found by taking $z = 0$ in the equation $x^2 + 4y^2 + z^2 = 4$, yielding $x^2 + 4y^2 = 4$, or the ellipse $\frac{x^2}{4} + y^2 = 1$ in the xy-plane.
The surface is given by $z = f(x,y) = \sqrt{4 - x^2 - 4y^2}$, so, taking our partial derivatives:
$f_x = \frac{1}{2}\left(4 - x^2 - 4y^2\right)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{4 - x^2 - 4y^2}}$
$f_y = \frac{1}{2}\left(4 - x^2 - 4y^2\right)^{-\frac{1}{2}}(-8y) = \frac{-4y}{\sqrt{4 - x^2 - 4y^2}}$

Therefore, the following is an integral representing the surface area of the top half of the ellipsoid:
$$\int_{-2}^{2}\int_{-\sqrt{1-\frac{x^2}{4}}}^{\sqrt{1-\frac{x^2}{4}}} x^2z\sqrt{\frac{x^2}{4 - x^2 - 4y^2} + \frac{16y^2}{4 - x^2 - 4y^2} + 1}\, dy\, dx$$

17. Use Green's Theorem to evaluate $\oint_C (y^3 + \sin(x^2))\, dx + (x^3 + \cos(y^2))\, dy$, where C is the circle $x^2 + y^2 = 4$ traversed counterclockwise.
Solution:

Recall that by Green's Theorem, $\oint_C (y^3 + \sin(x^2))\, dx + (x^3 + \cos(y^2))\, dy = \iint_R N_x - M_y\; dA$.

Here, $M_y = 3y^2$, $N_x = 3x^2$, and R is the circle of radius 2 centered at the origin.

Then we have $\iint_R 3x^2 - 3y^2\; dA$, which we translate into polar coordinates:
$$\int_0^{2\pi}\int_0^2 \left(3(r\cos\theta)^2 - 3(r\sin\theta)^2\right) r\, dr\, d\theta = \int_0^{2\pi}\int_0^2 3r^3\cos^2\theta - 3r^3\sin^2\theta\, dr\, d\theta$$
$$= \int_0^{2\pi} \frac{3}{4}r^4\left[\cos^2\theta - \sin^2\theta\right]\Bigg|_0^2 d\theta = \int_0^{2\pi} \frac{3}{4}(16)\left(\cos^2\theta - \sin^2\theta\right) d\theta$$

Recall: $\cos^2\theta - \sin^2\theta = \cos(2\theta)$, so we have:

$$= \int_0^{2\pi} 12\cos(2\theta)\, d\theta = 6\sin(2\theta)\Bigg|_0^{2\pi} = 0$$

18. Let $\vec{F} = \langle y^2 + x,\, y + xz,\, x\rangle$ and S the sphere $x^2 + y^2 + z^2 = 1$. Use the Divergence Theorem to find $\iint_S \vec{F} \cdot \vec{n}\, dS$, where $\vec{n}$ is the outward normal to S at (x, y, z).

By the Divergence Theorem, $\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_Q \nabla \cdot \vec{F}\ dV$.

Here, $div(\vec{F}) = \nabla \cdot \vec{F} = 1 + 1 + 0 = 2$

Thus we have: $\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_Q 2\ dV$

Since the solid enclosed by the surface is a shere, using a familiar geometric formula: $= 2\frac{4}{3}\pi(1)^3 = \frac{8\pi}{3}$.

19. Use Stokes' Theorem to evaluate $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$, where S is the portion of $z = \sqrt{4 - x^2 - y^2}$ above the xy-plane, with $\vec{n}$ upward, and $\vec{F} = \langle zx^2,\, ze^{x+y} - x,\, x\sin(y^2)\rangle$.

Solution:

Since S is given by the portion of $z = \sqrt{4 - x^2 - y^2}$ above the xy-plane, S is a hemisphere with radius 2. Therefore, the boundary of the surface is a circle of radius 2 in the xy-plane, which is a simple closed curve.

Since we are using the upward normal vector, from the righthand rule, we orient give the circle an anticlockwise orientation. This curve has parameterization:

$$C : \begin{cases} x(t) = 2\cos t \\ y(t) = 2\sin t \quad 0 \le t \le 2\pi \\ z(t) = 0 \end{cases}$$

By Stokes' Theorem, $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot d\vec{r} = \oint_C zx^2\, dx + ze^{x+y} - x\, dy + x\sin y^2\, dz$

$$= \int_0^{2\pi} 0(4\cos^2 t)(-2\sin t) + [(0)e^{2\cos t + 2\sin t} - 2\cos t](2\cos t) + (2\cos t\sin(4\sin^2 t))(0)\ dt = \int_0^{2\pi} -4\cos^2 t\ dt$$

$$= \int_0^{2\pi} -2 - 2\cos 2t\, dt = -2t - \sin 2t\Big|_0^{2\pi} = -4\pi$$