Math 102
Alternate Forms of the Conditional and Logical Arguments

Conditional Statements:

Since conditional statements (statements of the form $p \to q$) are used to describe "cause and effect" relationships, they play a crucial role written communication and in logical argumentation.
Because of the importance of conditional statements, we need to be able to recognize when a statement is conditional in form. It is not always as simple as looking for "If ... Then" statements. There are several other ways of expressing conditionals in written English. We must be careful to correctly identify which part of the statement is the hypothesis (the p or "cause" part of the statement) and which part is the conclusion (the q or "effect" part of the statement).
Here is a list of some common ways to express conditional statements in written English. In all of these, p is the hypothesis and q is the conclusion. Knowing these alternative forms will be helpful in translating from English to the correct symbolic logical form.

- If p then q
- q if p
- p only if q
- p is sufficient for q
- q is necessary for p
- q whenever p

Forms Related to the Conditional:

Given a conditional statement with hypothesis p and conclusion q, there are three related "conditional type" forms that can be formed based on the original conditional statements. They are as follows:

- **Conditional:** $p \to q$
- **Converse:** $q \to p$
- **Inverse:** $\sim p \to \sim q$
- **Contrapositive:** $\sim q \to \sim p$

Example: Given the conditional: "If I get a flat tire then I will be late for work.", we can write three related forms:

- **Converse:** $[q \to p]$ If I am late for work then I got a flat tire.
- **Inverse:** $[\sim p \to \sim q]$ If I do not get a flat tire then I will not be late for work.
- **Contrapositive:** $[\sim q \to \sim p]$ If I was not late for work then I did not get a flat tire.

An natural and important question to address is whether or not these forms are logically equivalent to the original conditional statement. Thinking through the example above, which, if any, of these three statements seem to be logically equivalent to the statement "If I get a flat tire then I will be late for work."?

To determine this precisely, we should build the truth tables for these expressions and compare them to the truth table for a standard conditional statement:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

From this, we see that the *contrapositive* statement **is** logically equivalent to the original conditional. The *converse* and the *inverse* are logically equivalent to each other but are **not** equivalent to the original conditional statement.

Logical Arguments:

Consider the following example of a logical argument:

Premise 1:	If I go outside without a coat, then I will catch a cold.
Premise 2:	I did not catch a cold.
Conclusion:	Therefore, I did not go outside without a coat.

The primary question we would like to address is whether or not the argument presented above is logically valid. That is, given that the stated premises are true, do the basic principles of logic demand that the conclusion is also true?

In general, we would like to find a way to determine whether or not any argument that we are presented with is logically valid.

- A **logical argument** is a series of statements called **premises** followed by a single statement called the **conclusion**.
- In an argument, all of the premises are assumed to be true statements (although on other levels, arguing premises can be important, but we will not look at this part of the process).
- An argument is **valid** if the conclusion follows from the truth of the premises. That is, if whenever the premises are true, then the conclusion is also true.

Using Truth Tables to Verify Logical Arguments:

One way to check the validity of a given argument is to use the truth table method. The steps in this method are as follows.

- First, we define variables and translate each of the premises and the conclusion statement into symbolic form.
- Next, we combine these into a single symbolic statement of the form:
  (First premise) $\wedge$ (Second premise) $\wedge$ ... $\wedge$ (Last premise) $\rightarrow$ (Conclusion).
- Finally, we build the truth table for the combined statement. If this statement is a tautology [that is, if every entry in the final column is T] then the original argument is **valid**. Otherwise, the argument is **invalid**.

Let's carry out this process on out original example to check to see if the argument is valid or invalid.

- We begin by choosing the variables g : "I go outside without a coat." and c : "I catch a cold.". With these variables, the original argument becomes:

Premise 1:	$g \rightarrow c$
Premise 2:	$\sim c$
Conclusion:	$\therefore \sim g$

- Next, we combine these to form a single statement:
  $(g \rightarrow c) \wedge (\sim c) \rightarrow \sim g$

- Finally, we build the truth table for our combined expression:

g	c	$g \rightarrow c$	$\sim c$	$(g \rightarrow c) \wedge \sim c$	$\sim g$	$(g \rightarrow c) \wedge (\sim c) \rightarrow \sim g$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

  Since the last column of this table is all T's, we can conclude that this is a **valid** argument.

  Notice that verifying this argument had nothing to do with the specifics of what each statement in the argument said. The validity of the argument is based solely on the form of the argument and the basic principles of logic. Because of this, if we were to run across an argument that had the same form in the future, its truth table would look exactly the same as this one, so it would automatically be valid.