Math 102
Counting Methods

Counting has to do with finding the number of objects present (or options available) in a particular situation. In this Chapter, we will learn several counting methods.

The Listing Method: One of the most straightforward ways to count is to list all of the objects one at a time.

Examples: Use the listing method to count the number of objects in each of the following situations.

1. How many outcomes are possible when flipping a coin twice and recording the results?

2. How many outcomes are possible when flipping a coin three times and recording the results?

3. How many outcomes are possible if a six sided die is rolled and the result is recorded? What if the die is rolled twice?

Note: If there are a large number of outcomes, using the listing method becomes tedious and inconvenient.

Tree Diagrams: A second counting method we can use is to draw a **tree diagram**. To make such a diagram, we first divide the situation we are counting into different steps. The first "level" or our tree diagram will have a **branch** for each option in that step. The next level adds second level branches to each of the first level branches - one second level branch for each option in the second step. We continue this process until all of the steps have been completed. The total number of objects in this counting situation is the number of branches added during the last step.

Examples: Use a tree diagram to find the number of outcomes in each of the following situations.

1. How many different outfits could you make if you own 10 shirts and 7 pairs of pants and an outfit consists of one shirt and one pair of pants?

2. How many outcomes are possible when flipping a coin four times and recording the results?

3. How many outcomes are possible if a six sided die is rolled *twice* and the results are recorded?
What if the die is rolled *three times*?

The Fundamental Counting Principle: The tree diagrams we used to solve the counting problem examples above illustrate an important principle. Suppose that a counting situation can be broken down into a series of separate tasks. If the first task con be carried out in a ways, the second task in b ways, the third task in c ways, and so on, then the total number of outcomes in this counting situation is $a \times b \times c \times ...$ That is, we find the total number of options by *multiplying* the number of possible ways to complete each task.

Slot Diagrams: Drawing tree diagrams takes up a lot of space and it gets to be difficult to draw in all of the branches. Another way to organize the information in a counting problem where the Fundamental Counting Principle applies is to use a slot diagram. In a slot diagram, we once again divide the counting situation into separate tasks. Then, we simply draw a blank for each step. Finally, we write the number of options available in each step in the blank or "slot" and multiply the numbers in each blank together to compute the total number of options.

Examples:

1. Suppose that you need to choose a 4-digit PIN for your credit card. How many different PIN numbers are possible?

2. How many different PIN numbers are possible of repetition is **not** allowed?

3. Suppose a state issues license plates to vehicles consisting of 3 letters followed by 3 numbers. How many different license plates are possible?

4. A postal zip code is a 5-digit number assigned to each postal delivery zone. Postal zip codes cannot start with a zero. How many different zip codes are there?

5. Suppose that 5 panelists (3 Biologists and 2 Chemists) are to be seated in a row with 5 chairs.

   (a) How many different ways could the panelists be seated?

   (b) How many ways could the panelists be seated if no two Biologists are seated next to one another?

   (c) How many ways could the panelists be seated if no Biologist sits in an "end" seat?