Math 102 Using Euler Diagrams to Verify Syllogisms involving Quantified Statements

- A syllogism is a set of statements called **premises** followed by a statement called a conclusion.
- A syllogism of **valid** if whenever the syllogism is true, the conclusion is also true. Otherwise, the syllogism is said to be **invalid**.
- We distinguish these syllogisms from the logical arguments we covered last section because we will allow the premises in these syllogisms to contain quantifiers ("all", "some", and "none" statements).
- Since our premises will involve quantifiers, we will use different methods to check their validity. Specifically, we will use Euler Diagrams.

Example 1: Use an Euler Diagram to determine whether or not the following syllogism is valid.

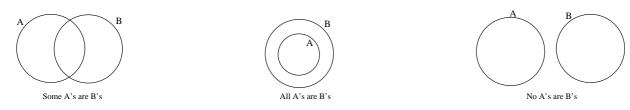
All cows eat grass

Fido does not eat grass

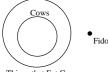
Therefore, Fido is not a cow

To construct a diagram to check the validity of a syllogism, we begin by reading the statements and carefully consider the classes of objects that are being related. Here, we have two classes of objects: "Cows" and "Things that Eat Grass". We also have Fido, which serves as an example of an object that is **not** is the "Things that Eat Grass" category.

In our diagram, we will use circles to represent classes of objects and dots to represent specific examples of objects either within out outside our categories. We must look at how the categories relate so we can tell how to draw the circles relationships with each other. Here are the three main possibilities:



With this in mind, we turn our attention back to our example and draw a diagram representing the situation described by our premises:



Things that Eat Grass

Notice that we drew Fido outside the "Things that Eat Grass" circle since our premises tell us that Fido is *not* in that category and we drew the "Cows" circle inside the "Things that Eat Grass" circle since all cows eat grass. From the digram, we can conclude that this syllogism is **valid**, since the relationships described by the premises force the conclusion to be true as well.

Example 2: Use an Euler Diagram to determine whether or not the following syllogism is valid.

Caviar is expensive.

All items sold at WalMart are not expensive.

Therefore, WalMart does not sell caviar.

Example 3: Use an Euler Diagram to determine whether or not the following syllogism is valid. Some movies are entertaining.Some movies make a profit.All entertaining movies make a profit.Saw VI made a profit.

Therefore, Saw VI was entertaining.

Example 4: Use an Euler Diagram to determine whether or not the following syllogism is valid. Winners never quit. Quitters never win. I did not quit. Therefore, I won.

Example 5: Given the following statements, draw an Euler diagram and use it to find a valid conclusion. Some A' are B's.
Some B's are C's.
No C's are A's.
All D's are C's.
Therefore ???