Math 102
Final Exam Practice Problems - Part 1

1. List the 7 problem solving strategies from section 1.1 in your book.
   (a) Draw Pictures
   (b) Choose Good Names for Unknowns
   (c) Be Systematic
   (d) Look for Patterns
   (e) Try a Simpler Version of the Problem
   (f) Guided Guessing (or - Guessing is OK)
   (g) Convert a New Problem in to an Older One

2. Determine whether each statement below is true or false. If it is true, explain how you know that it is true, and if it is false, give an example that shows it must be false.
   (a) If $A \cup \emptyset = A$, then $A = \emptyset$.
   False. $A \cup \emptyset = A$ for *any* set A.
   (b) If A and B are equivalent as sets, then A and B are also equal as sets.
   False. Equivalent sets have the same number of elements, while equal sets contain precisely the same elements.

3. (a) Use set notation to list all the elements of the set:
   $A = \{\ x \mid x$ is a whole number less than 40 that is evenly divisible by 3 $\}$
   Solution:
   $A = \{0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39\}$

   (b) Use set-builder notation to describe the set $\{2, 4, 8, 16, 32, 64, \ldots\}$
   Solution:
   $A = \{\ x \mid x$ is 2 raised to a positive whole number exponent $\}$

4. Given that $A = \{\ x \mid x$ is a letter in the word *apple* $\}$, $B = \{\ x \mid x$ is a letter in the word *please* $\}$, $C = \{\ x \mid x$ is a letter in the word *appeases* $\}$, and $D = \{\ \emptyset\ \}$, indicate whether the following are True or False (you do NOT need to justify your answers)
   (a) $\{a\} \in A$ False
   (b) $d \in A$ False
   (c) $\emptyset \subset A$ True
   (d) $A \subseteq B$ True
   (e) $B \subset C$ False
   (f) $B = C$ True
   (g) $D \subset B$ False
   (h) $\emptyset \in D$ True

5. Let $U = \{\ x \mid x$ is an odd number less than 30 $\}$; $A = \{1, 3, 5, 7, 11\}$; $B = \{5, 7, 11, 17, 21\}$; $C = \{3, 7, 9, 21, 23, 29\}$
   (a) Write U in roster form
   $U = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
   (b) $A - B = \{1, 3\}$
   (c) $A \cap C = \{3, 7\}$
   (d) $A' = \{9, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
   (e) $B \cup C =$
   $B \cup C = \{3, 5, 7, 9, 11, 17, 21, 23, 29\}$
   (f) $B - A = \{17, 21\}$
   (g) $n(A \cup C) = n(\{1, 3, 5, 7, 9, 11, 21, 23, 29\}) = 9$
   (h) $n(C \cap B') = n(\{3, 9, 23, 29\}) = 4$

6. (a) List all the subsets of the set $\{a, b, c, d\}$
   $\{a, b, c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b\}\{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset$
   (b) How many subsets does the set $\{a, b, c, d, e, f\}$ have?
   $\{a, b, c, d, e, f\}$ has $2^6 = 64$ subsets.

(c) Write out the first 7 rows of Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

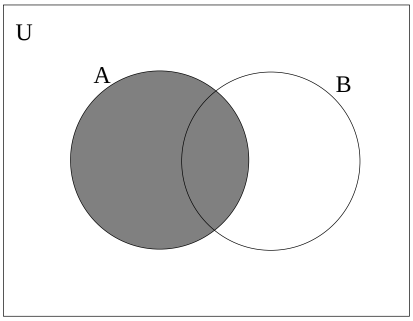
1 6 15 20 15 6 1

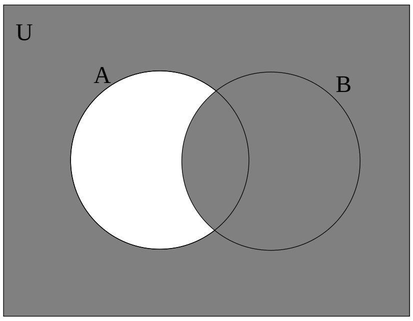
(d) Use Pascal's Triangle to find the number of 5 element subsets of $\{a, b, c, d, e, f\}$

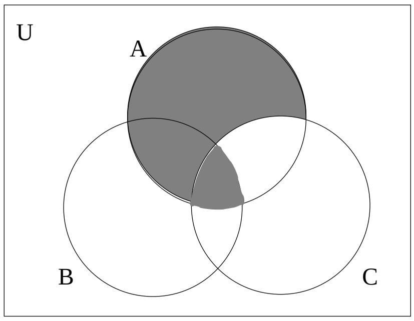
There are 6 5 element subsets of $\{a, b, c, d, e, f\}$.

7. Illustrate the following by shading the appropriate regions of the given Venn diagrams:

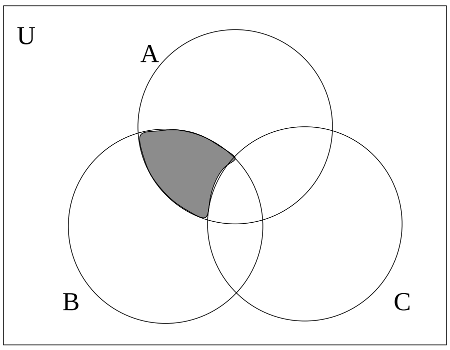
(a) $(A \cup B) - A'$ (b) $(A - B)'$ (c) $A \cap (B \cup C')$

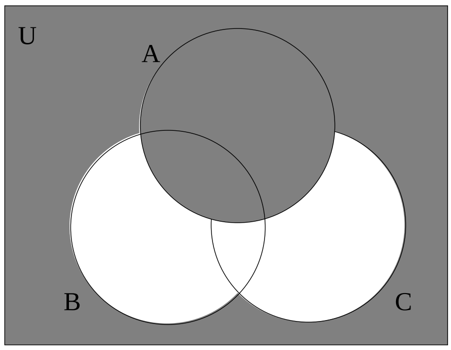






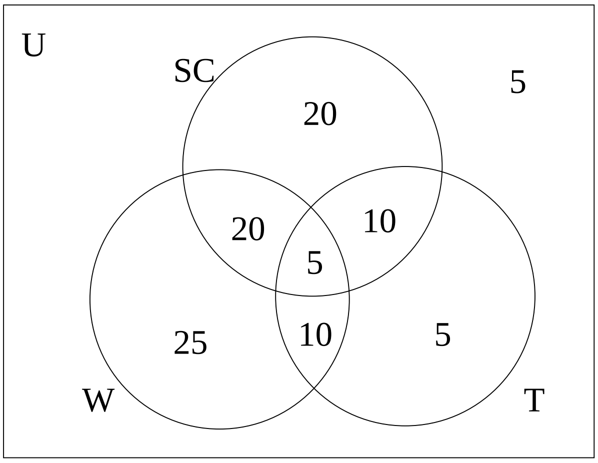
8. Use set notation to describe the shaded regions in each Venn diagram given below:





(a) $(A \cap B) - C$ (b) $A \cup (B \cup C)'$

9. A survey of 100 college students asked what they plan to do among the following 3 choices: Work, Take Summer Classes, and Travel. Suppose the survey found that 55 plan to take summer classes, 25 plan to work and take summer classes, 5 plan to do all three, 15 plan to both travel and take summer classes, 15 plan to travel but don't plan to work, 70 plan to take summer classes or travel, and 5 plan to do none of the three.



   (a) How many plan to travel but do not plan to take summer classes? 15
   (b) How many plan to travel and work this summer? 15
   (c) How many plan to take summer classes or work? 90
   (d) How many plan to travel? 30
   (e) How many plan to work but not travel or take classes? 25

10. For each of the following, state whether the situation is an example of inductive or deductive reasoning:

   (a) You notice that you are tired all morning if you don't have breakfast, so you decide to start eating breakfast every morning before you leave to go to school.
   Inductive reasoning
   (b) After getting your monthly paycheck, you do some computations to make sure that your employer paid you at the time-and-a-half rate for the 7 hours of overtime you put in during one of the weeks that month.
   Deductive reasoning

11. Use inductive reasoning to predict the next two terms in each of the following sequences:

   (a) $20, 19, 17, 14, 10, ...$
   Next two terms: 5, -1 (the pattern is subtracting number larger each time $20 - 1 = 19, 19 - 2 = 17$, etc.)
   (b) $1, 4, 3, 8, 5, 12, 7, 16, ...$
   Next two terms: 9, 20 (hint: look at the even terms and odd terms as separate sequences woven together)

12. Determine whether or not each of the following are statements:

   (a) I never missed class this semester. - Statement.
   (b) How many times did I miss class this semester? - Not a Statement.

13. Negate each of the following statements, then rewrite them as English sentences:

    (a) All of my friends have already left for the summer.
    It is not the case that all my friends have already left for the summer.
    At least one of my friends has not yet left for the summer.

    (b) This summer I will not get a job or I will not take classes.
    It is not true that this summer I will not get a job or I will not take classes.
    I will get a job this summer and I will take classes this summer.

14. Given p : cows like grass, q : pigs like slop, r : horses like oats, and s : mice like cheese, translate the following statements into words:

    (a) $p \vee (\sim r)$
    Cows like grass or horses do not like oats.

    (b) $(r \wedge s) \vee (\sim p)$
    Horses like oats and mice like cheese, or cows do not like grass.

    (c) $(p \wedge (\sim s)) \rightarrow q$
    If cows like grass and mice do not like cheese, then pigs like slop.

15. Explain, in your own words, the difference between "exclusive or" and "inclusive or"

    Exclusive or is used to indicate that one of two things is true, but not both.

    Inclusive or indicates that one of two things are true, or both could be true as well.

16. Given the statements: p : I win the lottery, and q : I quit going to school

    (a) Write the conditional statement relating p to q in words.
    If I win the lottery then I will quit going to school.

    (b) Write the converse in words.
    If I quit going to school, then I won the lottery.

    (c) Write the inverse in words.
    If I do not win the lottery, then I will keep going to school.

    (d) Write the contrapositive in words.
    If I keep going to school, then I did not win the lottery.

    (e) Indicate which of these statements above are logically equivalent to each other. You do not need to prove your answer.
    The conditional and the contrapositive are logically equivalent.
    The converse and the inverse are logically equivalent.

17. Given that p is false, q is true, r is false, and s is false:

    (a) What is truth value of the statement: $(p \vee q) \rightarrow (\sim r \wedge s)$

p	q	r	s	$p \vee q$	$\sim r$	$\sim r \wedge s$	$(p \vee q) \rightarrow (\sim r \wedge s)$
F	T	F	F	T	T	F	F

    Therefore, with these truth values, the logical expression is False

    (b) How many rows would the full truth table for the expression $(p \wedge s) \rightarrow (r \vee \sim s)$ have?
    Since there are 3 variables, there will be $2^3 = 8$ rows in the full truth table for this expression.

18. Identify the form of the following arguments, and state whether the given argument is valid:

   (a) I will go to the store to buy milk or I will make do without milk today. I did not make do without milk today. Therefore, I went to the store to buy milk.

   We define p : I go to the store and buy milk, and q : I will make do without milk today.

   Then the argument has the form:

   $$\begin{array}{l} p \vee q \\ \underline{\sim q} \\ \therefore p \end{array}$$

   This is a Disjunctive Syllogism
   Therefore, this argument is valid

   (b) If I get over this cold, then I will go out with my friends on Saturday night. I did not get over this cold. Therefore, I did not go out with my friends on Saturday night.

   We define p : I get over this cold, and q : I go out with my friends on Saturday night.

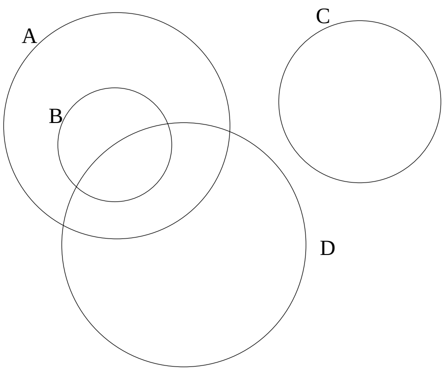
   Then the argument has the form:

   $$\begin{array}{l} p \rightarrow q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$$

   This is the Fallacy of the Inverse
   Therefore, this argument is invalid

19. (a) (4 points) Draw an Euler diagram for the statements: "All B's are A's", "All A's are not C's", and " Some D's are B's"

   One possible Euler diagram is:

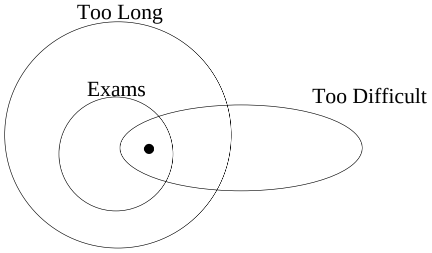


   (b) State two different valid conclusions that can be made based on the statements in part (a) above.

   1. Some D's are A's
   2. No B's are C's

20. Use Euler diagrams to determine whether the following syllogism is valid or invalid:

   All exams are too long.
   Some exams are too difficult.
   Therefore, some exams are too long and too difficult.



Notice that this syllogism is Valid.

21. Use a truth table to determine whether or not the following argument is valid:

If I study, then I will get a good grade on the final exam.
If I do not get a good grade on the final exam, then I will not pass this class.
I passed this class.
____
Therefore, I studied.

First, we need to translate the argument into logical symbols. To do this, we take p : I study, q : I get a good grade on the final exam, and r : I pass this class.

With these variables, the form of this argument is:

$$\begin{array}{l} p \to q \\ \sim q \to \sim r \\ r \\ \hline \therefore p \end{array}$$

With this symbolic representation, to assess the validity of this argument, we need to investigate the logical expression: $(p \to q) \wedge (\sim q \to \sim r) \wedge r \to p$

p	q	r	$p \to q$	$\sim q$	$\sim r$	$\sim q \to \sim r$	$(p \to q) \wedge (\sim q \to \sim r) \wedge r$	$(p \to q) \wedge (\sim q \to r) \wedge r \to p$
T	T	T	T	F	F	T	T	T
T	T	F	T	F	T	T	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	T	T	F
F	T	F	T	F	T	T	F	T
F	F	T	T	T	F	F	F	T
F	F	F	T	T	T	T	F	T

Notice that the last column of the truth table has a False entry. Therefore, this argument is not valid.

22. Given the argument:

$$\begin{array}{l} p \to (q \wedge r) \\ s \vee \sim r \\ t \to \sim s \\ t \\ \hline \therefore \sim p \end{array}$$

Fill in the missing statements and reasons in the following two column proof:

1. t	Premise
2. $t \to \sim s$	Premise
3. $\sim s$	1, 2, Law of Detachment
4. $s \vee \sim r$	Premise
5. $\sim r$	3, 4, Disjunctive Syllogism
6. $\sim q \vee \sim r$	5, Addition
7. $\sim (q \wedge r)$	6, DeMorgan's Law
8. $p \to (q \wedge r)$	Premise
9. $\sim p$	7, 8, Law of Contraposition